DOCUMENT RESUME

ED 043 508 24 SE 009 578

AUTHOR	Suydam, Marilyn N.
TITLE	Interpretive Study of Research and Development in Elementary School Mathematics: Phase 2, Final Report.
INSTITUTION	Pennsylvania State Univ., University Park.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO	BR-8-0586
PUB DATE	31 Aug 70
GRANT	OEG-0-9-480586-1352(010)
NOTE	179p.
EDRS PRICE	EDRS Price MF-$0.75 HC-$9.05
DESCRIPTORS	Curriculum, *Educational Research, *Elementary School Mathematics, Instruction, Mathematics, *Mathematics Education, Research, *Research Reviews (Publications)

ABSTRACT

The Interpretive Study of Research and Development in Elementary School Mathematics was designed to attack the problem of applying research in the classroom by providing a compilation of the research and a synthesis of the findings. The purpose of Phase II of the Study was to develop materials which could be readily used and disseminated. A set of eleven bulletins and a set of five films were developed for use by college teachers of courses on the teaching of elementary school mathematics. This report concentrates on the development, dissemination, and evaluation of these bulletins and films. Copies of all the bulletins and scripts used for all the films are included in the appendices bound with the report. One conclusion drawn from the reactions of those receiving the developed materials is that there is a continuing need to provide a readable, readily available source of information on research related to teaching and learning elementary school mathematics. (FL)

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ED043508

FINAL REPORT
Project No. 8-0586
Grant No. OEG-0-9-480586-1352(010)

INTERPRETIVE STUDY OF RESEARCH AND DEVELOPMENT IN ELEMENTARY SCHOOL MATHEMATICS: PHASE II

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August 31, 1970

The research reported herein was performed pursuant to a grant with the Office of Education, U.S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy.

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PREFACE

It is impossible to thank all of the people who have helped with this Interpretive Study, for such a list would have to include all of those in schools and colleges who have given encouragement and reactions. Nevertheless, the vital roles which some took deserve special recognition.

J. Fred Weaver, Professor of Education at the University of Wisconsin-Madison, served as the consultant who aided in the development, writing, and editing of the materials: the results of his efforts are evident, and very appreciated. Thomas E. Kieren, now Associate Professor of Education at the University of Alberta, directed the development of the film on mathematics laboratories and also answered questions about a variety of topics: others will also thank him when they see the film.

Lynn Pearson, Patricia Lazar, Judith Bechtel, Mary Alice McCabe, and Robert MacLean served as teachers in the films: they and the children who starred deserve special thanks. Florence Hammonds was the exceptionally capable graduate assistant who served as "stage manager" for the films, while John Howell and Richard Kohr aided in planning mailing procedures and processing data. Beverly Brooks served as administrative assistant, very capably assuming responsibility for processing the bulletins. James D. Gates, Executive Secretary of the National Council of Teachers of Mathematics, William L. Pharis, Executive Secretary of the National Association of Elementary School Principals, and James Kovach, also of NAESP, provided the mailing lists so necessary for the success of this project.

To all others who helped: thank you, too!

I. OVERVIEW OF PROJECT

A. Need for the Study

Research on elementary school mathematics has assumed increased importance during the past decade, as have most areas of educational research. With more research being done, there is a greater need to synthesize the resulting body of knowledge so that researchers can consider what has been done as they design future research. There has also been an increasing emphasis on applying research in the classroom, integrating the results into both curriculum development and lesson planning.

The findings of educational research have not had the impact on curriculum decision-making in elementary school mathematics that they could have had. Findings have not been readily available; for many topics, they are equivocal or conflicting; reports are not always written in language which is clear to the non-researcher; the applicability of results to a specific situation is unclear.

The Interpretive Study of Research and Development in Elementary School Mathematics was designed to attack these problems by providing (1) a compilation of the research and (2) a synthesis of the findings. The resulting products comprise a study of the status of research on elementary school mathematics.

B. Background, Phase I

The Interpretive Study was developed in two phases. These objectives were reached during Phase I:

(1) Reports of research on elementary school mathematics through 1968 were collected, analyzed, categorized on ten aspects, annotated, and evaluated. These were collated with reports collected in a previous project,[1] to form a total pool of 1050 reports of research on elementary school mathematics.

(2) Dissertations from 1966 through 1968 were listed, categorized by topic, and annotated. A list of dissertations completed prior to

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[1] Suydam, Marilyn N. An Evaluation of Journal-Published Research Reports on Elementary School Mathematics, 1900-1965. Unpublished doctoral dissertation, The Pennsylvania State University, 1965.

1966 was already compiled.[2] The complete pool of dissertations numbers approximately 700.

(3) Representatives of specified target audiences were contacted. They supplied a list of questions to which they hoped answers could be supplied from research.

(4) Summaries of the research were written in response to these and other pragmatically derived questions. A list of the most applicable findings of research was also developed.

(5) Ten major curriculum development projects were visited, and interviews taped with the directors to provide explicit information on the background, progress and status of these. Reports of other projects, including those documented in the Educational Resources Information Center (ERIC), were annotated.

(6) In a summary chapter, key research and developmental trends were discussed.

The Final Report[3] for Phase I consists of three volumes. Volume 1 describes the study and presents the summarized findings. Volume 2 contains the compilation of categorized research reports. In Volume 3, reports of developmental projects are summarized and interviews with project directors are included. (For a summary of an evaluation by a sample of the users of these materials, please see pages 10-17.

C. Purpose, Phase II

The need for materials which could be readily used and disseminated was evident. During Phase II, two types of materials were developed: a set of eleven bulletins and a set of five films. The bulletins have been and are being disseminated to two primary target audiences: (1) college teachers of courses on the teaching of elementary school mathematics, and (2) elementary school principals. Each of these audiences deals with a sizeable number of those who form the true audience, the teacher in the elementary school classroom. Two subsets of this set are being reached, to some extent: the pre-service teacher, whose influence

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[2] Ibid.

[3] Suydam, Marilyn N. and Riedesel, C. Alan. Final Report of the "Interpretive Study of Research and Development in Elementary School Mathematics." U.S.O.E. Grant No. OEG-0-9-480586-1352(010), The Pennsylvania State University, June 30, 1969.

will be felt in the years to come, and the in-service teacher, whose influence is immediate and continuing.

The information which follows in this report concentrates on the development, dissemination, and evaluation of these materials. The Schedule of Activities for Phase II, presented on Figure 1, indicates the time line which was followed.

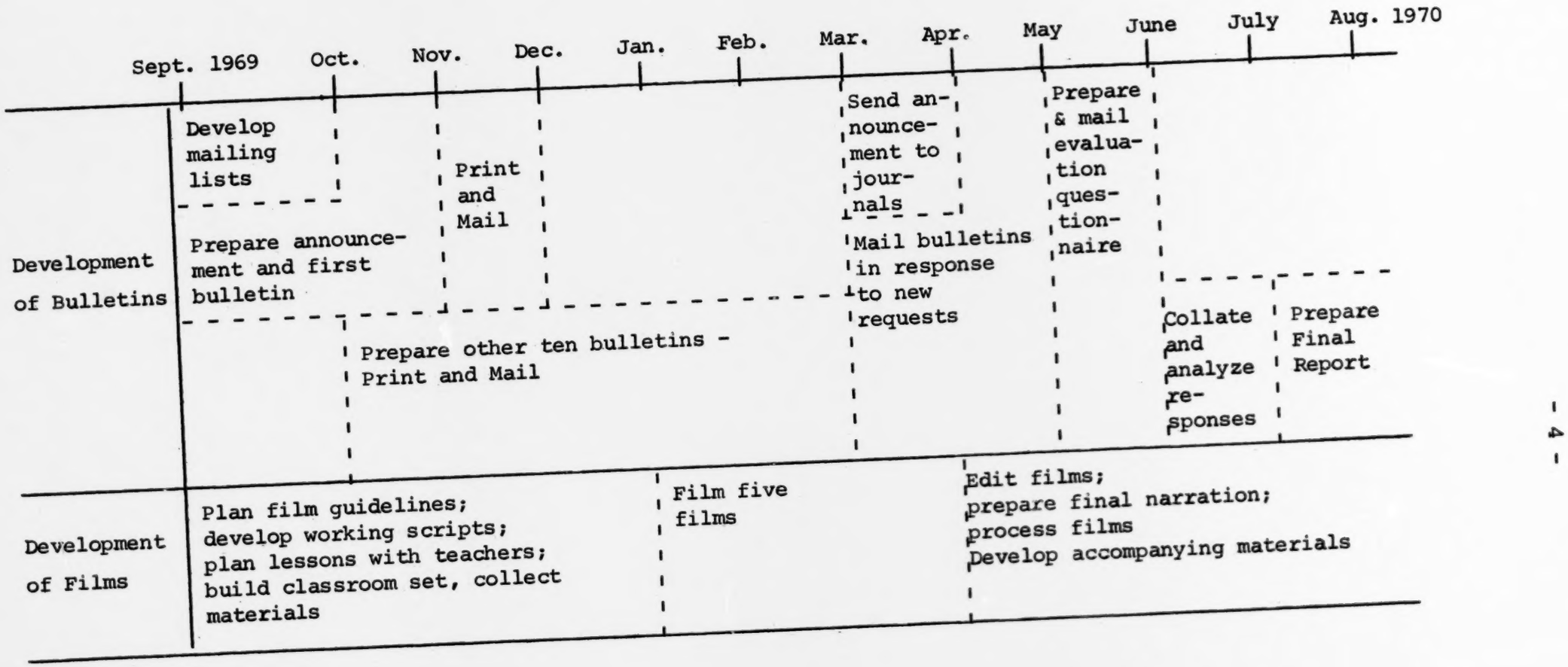

FIGURE 1

SCHEDULE OF ACTIVITIES FOR PHASE II

II. DEVELOPMENT OF MATERIALS

A. Bulletins

In a planning session during the first week of September, the format, structure, and content of the set of bulletins was determined. While it was proposed originally that only a letter be sent to those on the mailing lists, it was decided that inclusion of the first bulletin with this letter would provide a specific illustration of the type of material to be developed. A copy of this letter is included in Appendix A, followed by the bulletins.

The format and structure of the bulletins was determined with the needs of the user-groups as the primary consideration. It was decided that the first page (front and back) would be printed with a perforated edge, to facilitate distribution to teachers. It was titled "Overview," and is a summary of the material which is discussed in greater depth in the remainder of the bulletin, "A Closer View." Thus there is material both for those who, with limited time, want a brief synthesis, and for those who want further elaboration of the research. In the "Overview," no references are cited, while in the "Closer View" specific research reports are cited. A list of references is included for further study.

It was decided that the bulletins would be structured around questions which were frequently asked about elementary school mathematics. When there was no answer to such a question, this would be stated. It was agreed that the material should be factual, referenced to actual research findings; opinions should be kept to a minimum and stated as opinions, not fact. That the material should be written as simply as possible, with terminology explained when appropriate, was a firm guideline followed in the preparation of all bulletins.

The topics for the eleven bulletins were determined by the importance of a topic to teachers and by the amount of research available on a topic. The topics which met these criteria and on which bulletins were written are:

A-1 Attitudes and Interests
A-2 Planning for Instruction
A-3 The Teaching-Learning Process
A-4 Individualizing Instruction
A-5 Instructional Materials and Media
A-6 Planning for Research in Schools

B-1 Addition and Subtraction with Whole Numbers
B-2 Multiplication and Division with Whole Numbers
B-3 Fractions and Decimals
B-4 Other Mathematical Topics
B-5 Verbal Problem Solving

The bulletins were written during the period between September and March. The preparation of each involved a series of stages:

(1) Structuring (determining questions and studies to be included)

(2) Writing of first draft

(3) Editing

(4) Revising

(5) Typing of final copy

(6) Printing

The first bulletin with the announcement letter was mailed in late November. Other bulletins were mailed to the respondees at approximately two-week intervals between January and April.

B. Films

Guidelines for the films were developed in September, and focus and format for each film was determined. It was agreed that each film would include illustrations of research findings being applied in a classroom, across a variety of content areas and grade levels. The titles for the films were selected:

Film 1. Using a Mathematics Laboratory Approach
Film 2. Using Diagnosis in a Mathematics Classroom
Film 3. Operations with Whole Numbers
Film 4. Practicing Mathematics Skills
Film 5. Problem Solving Techniques

During the months that followed, the planning became increasingly specific. Working scripts were developed, setting the order of classroom sequences and other scenes. Lessons were planned with the teachers who would be filmed with their pupils. Materials were collected, and a classroom set constructed.

Filming began in February and was completed in April, after which the films were edited and narration written. In Appendix B is a general description of the scenes in each film, and the narration which accompanies it.

Supplementary materials for use in discussion and study with each film were developed. These are included in Appendix C.

III. DISSEMINATION

A. Bulletins

1. Announcement Data

College teachers of courses on the teaching of elementary school mathematics were identified by collating lists from The National Council of Teachers of Mathematics, which provided tables for 1300 mathematics educators at the college level, and from those colleges and universities preparing elementary school teachers. A randomly selected list of 8500 elementary school principals was provided by the National Association of Elementary School Principals.

A letter describing the proposed set of bulletins and enclosing Bulletin A-1, "Attitudes and Interests" (see Appendix A) was sent to approximately 8600 persons (2100 college professors and 6500 principals). If the recipient wanted to receive the remaining bulletins, he had to return a form to the Project. Space was provided for other names.

An announcement (see Appendix D) about the availability of the bulletins was submitted to fourteen publications. To date it has appeared in American Education (July 1970, page 22), "Bulletin for Leaders," NCTM (June 1970, page 2), Grade Teacher (September 1970), and National Elementary Principal (May 1970, page 85), and other journals have indicated that it will be published.

2. Distribution Data

Table I indicates the number of bulletins mailed to initial target audiences in each state. The number of persons requesting the bulletins has since increased, as others received information about the materials from Project staff (at conferences), from announcements in journals, or from colleagues receiving the bulletins. These requests are continuing to be received and processed at the rate of approximately 400 per month.

The bulletins and excerpts from them have appeared in a variety of publications, including those from several Title III centers and Croft Publications. This has also resulted in additional requests for copies.

TABLE I

DISTRIBUTION OF BULLETINS BY STATES
(as of June 1970)

State	Number of Respondees	State	Number of Respondees
Alabama	25	New Jersey	214
Alaska	10	New Mexico	5
Arizona	6	New York	232
Arkansas	10	North Carolina	49
California	124	North Dakota	10
Colorado	19	Ohio	157
Connecticut	210	Oklahoma	15
Delaware	7	Oregon	27
District of Columbia	20	Pennsylvania	256
Florida	45	Rhode Island	48
Georgia	76	South Carolina	15
Hawaii	15	South Dakota	7
Idaho	6	Tennessee	63
Illinois	91	Texas	42
Indiana	108	Utah	11
Iowa	55	Vermont	38
Kansas	37	Virginia	53
Kentucky	11	Washington	20
Louisiana	31	West Virginia	24
Maine	67	Wisconsin	91
Maryland	77	Wyoming	1
Massachusetts	355		
Michigan	150	International	
Minnesota	56	Brazil	1
Mississippi	8	Canada	27
Missouri	72	Canal Zone	1
Montana	9	Chile	1
Nebraska	19	Israel	1
Nevada	3	Micronesia	2
New Hampshire	26	TOTAL	3159

B. Films

The films are available for use after September 1, 1970. A notice about the availability of the films (see Appendix A) was sent to those on the mailing list in August 1970. They may be secured from The Pennsylvania State University and from The National Audio-Visual Center.

C. Conference Presentations

Presentations about the Project have been made at national annual meetings of the National Council of Teachers of Mathematics, the American Educational Research Association, and the National Association of Elementary School Principals. In addition, presentations were made at state-level meetings of these organizations.

During the coming year, it is anticipated that additional presentations will be made at NCTM, AERA, and NAESP meetings.

IV. EVALUATION

A. Phase I Questionnaires

An Evaluation Questionnaire (see Appendix E) was mailed during June 1970 to 160 educators who had received the Final Report for Phase I of the Interpretive Study. The responses received from 68 persons who returned the form by August 1 are presented on Table II. It should be noted that the reactions were generally favorable: 71% report that they have found it very helpful, while 26% report it is somewhat helpful. Volumes I and II were used most frequently. Over 500 others have used the 68 copies of the report in addition to the receiver; these include both pre-service and in-service teachers, as well as graduate students.

For some questions, comments and suggestions were requested. Responses to question 8, "How have you used the Final Report?," indicate that it is primarily a reference tool. Among the uses stated are:

(1) To locate references on a given topic

(2) To check research on a topic

(3) Suggested as independent study material or recommended reference

(4) In preparing for class lectures, discussions, and "talks"

(5) In preparing reports and summaries

(6) To update and refresh my knowledge of research in the field

(7) To update graduate students, calling attention to research about which they should be aware

(8) To provide background information

(9) To learn more about specific research

(10) For suggestions on teaching methods

(11) In curriculum planning

(12) In reviewing and evaluating our own research work

(13) For information!

Most indicated in response to question 9 that the summaries in Volume I were useful, noting that they were "brief and to the point," useful as a "guide in selecting items for more complete study," give "interesting comments for debate!," "concise and clear," and "easy to scan and abstract essentials." One college professor noted that "they vary in quality and scholarship," and this might have been noted by others. Those who did not find the summaries useful commented: "They don't adequately allow for flaws in research," "they are of necessity too brief," "I prefer more reporting of actual content of research," "the

TABLE II

SUMMARY OF DATA ON EVALUATION QUESTIONNAIRE FOR FINAL REPORT, PHASE I

Identification of categories of respondees used in column 1 on the table:

1. College teachers of mathematics methods courses
2. College teachers of mathematics content courses
3. College teachers of other courses
4. College teachers of both mathematics methods courses and mathematics content courses
5. College teachers of both mathematics methods courses and other courses
6. College teachers of both mathematics content courses and other courses
7. Other positions at college level
8. Principals
9. Classroom teachers
10. Mathematics coordinators/supervisors
11. Curriculum specialists
12. Undergraduate students
13. Graduate students
14. Others

The questions and answers are, of course, abbreviated: for the exact wording, please refer to the questionnaire in Appendix E.

TABLE II(CONTINUED)

Respondee Categories	Number of Respondees	Qu. 2: Frequently Use	Never Use	Occasionally Use	Qu. 3: Very Helpful	Somewhat Helpful	Not Helpful	Qu. 4: Vol. Most Used 1	2	3	Qu. 5: Vol. Least Used 1	2	3	Qu. 6: Others Used Yes	How Many	No	Qu. 7: Use With Pre-service	Graduate	In-service	Other
1	17	15	0	2	12	5	0	12	12	4	2	1	11	15	244	2	7	16	8	2
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	6	3	0	3	4	1	0	1	4	0	1	1	3	5	47	1	1	4	0	1
4	11	6	0	5	9	2	0	7	5	2	2	3	5	9	50	2	7	6	4	1
5	4	4	0	0	3	1	0	4	1	0	0	1	3	4	57	0	2	3	2	1
6	1	1	0	0	1	0	0	0	1	0	0	0	0	1	30	0	0	1	0	1
7	1	0	0	1	1	0	0	0	1	0	1	0	0	0	0	1	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	2	0	0	2	1	1	0	1	1	0	0	1	1	1	0	1	0	0	2	0
10	6	2	0	4	2	3	0	5	2	2	0	0	3	4	7	2	3	2	3	0
11	6	2	0	4	3	3	0	3	2	1	0	3	3	6	21	0	1	2	3	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	6	4	0	2	5	1	0	3	2	1	2	1	3	4	31	2	2	2	1	3
14	8	2	0	5	7	1	0	6	4	3	0	2	4	7	22	1	1	0	1	5

TABLE II (CONTINUED)

Respondee Categories	Qu. 8:	Qu. 9:			Qu. 10:	Qu. 11:	Qu. 12: Vol. 2		Qu. 13: Dissertation				Qu. 14:	Qu. 15:
	Number	Useful	Not Useful	Bulletins Better	Number	Number	Useful	Not Useful	Have	Don't Have	Send	No	Number	Number
1	16	13	2	7	14	1	16	0	3	14	12	0	9	4
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	6	3	0	1	3	1	4	0	2	4	1	0	3	0
4	10	10	1	3	8	2	9	0	3	7	8	0	8	2
5	4	4	0	0	4	0	4	0	2	2	2	0	3	1
6	1	1	0	0	1	0	1	0	0	1	1	0	1	0
7	1	0	1	0	0	0	1	0	0	1	1	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	2	1	0	1	2	0	2	0	0	1	0	0	1	0
10	4	4	1	0	4	1	4	0	0	4	3	1	3	0
11	6	3	1	1	4	1	4	0	1	5	4	2	4	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	6	5	0	1	4	2	5	0	1	5	4	0	5	1
14	5	6	0	1	3	2	7	0	5	3	2	0	3	1

TABLE II (CONTINUED)

Respondee Categories	Qu. 16: Vol. 3 Useful	Qu. 16: Vol. 3 Not Useful	Qu. 17: Number	Qu. 18: Number	Qu. 19: Number
1	12	1	9	2	11
2	0	0	0	0	0
3	1	1	1	0	4
4	8	2	9	0	7
5	2	0	2	1	3
6	1	0	1	0	1
7	0	0	0	0	0
8	0	0	0	0	0
9	2	0	1	1	1
10	2	1	2	0	4
11	5	1	4	1	2
12	0	0	0	0	0
13	5	1	3	2	4
14	7	0	3	1	4

overgeneralization decreases its value - I fear that many readers will accept this summary as fact, even though you have taken great pains to caution them of this danger." A number of persons thought the bulletins for Phase II were better, generally reflecting one comment that they are "more comprehensive - in a more convenient form."

The uses of Volume I cited in response to question 10 generally echoed the responses to question 8, as did the uses of Volume II requested by question 14. Other comments (questions 11 and 15) included:

(1) Excellent

(2) Excellent bibliography. I found this and all materials very useful and helpful.

(3) Index to topics would be helpful.

(4) I've referred to this material constantly since I've received it.

(5) To point out to higher officials in Washington that all that glitters is not gold . . .

(6) 'Answers' organized alphabetically would facilitate locating topics.

Comments on question 12 about the annotated lists in Volume II indicated that most who found them useful made comments such as: "They help me to quickly find reports in any area," they "bring much source information into one volume, hence facilitates research efforts," or they "give information about the type of research done." Only one person noted that the code is "awkward," while two commented that the evaluations or ratings were "interesting and helpful." Another (who had not received the bulletins from Phase II) suggested they "might be usefully summarized in a 'What Research Says' publication for wider distribution."

Most responses to question 16 indicated that the interviews in Volume III were useful because they "give a deeper insight into the philosophy and objectives of the people interviewed," "personalize certain complex opinions," "provide information not available elsewhere," and "give an awareness, an experience, a vicariousness of the realities." Several reactions were less favorable: "not always relevant," "interesting, but not much new."

Uses of Volume III (questions 17 and 18) included many mentioned on question 8, with many reflecting the idea that it is a "great help in simply being as current as it is." One suggestion was made that a "report of project details followed by interviews on essentially different features might be more useful."

The responses to question 19, "What suggestions do you have that would have improved the Final Report?," are varied. Among them are:

(1) I think the report represents a significant contribution in mathematics education. I only wish that it could be extended to include summary and analysis of studies in mathematics education at higher levels, i.e. high school and college.

(2) 'Improved': if it implies 'more useful to me,' then I wish it had been available sooner.

(3) None, really. It's a lovely job--thorough and useful. Someone should extend it to secondary level and beyond.

(4) I have thoroughly enjoyed examining your final report. It is and will continue to be useful.

(5) So far none--I have found them most helpful. The bulletins are an excellent idea and do hope you shall continue them.

(6) These reports represent a great contribution to the field of mathematics education. The bulletins growing out of this research are excellent. These bulletins would be more useful if available in large quantities for our students. You should be commended for a fine job--well-done!!

(7) One shouldn't quibble with a major task so well done. But (since you asked), I do believe that more uniformly high quality on the part of the students who worked on Volume I would have added validity to some interpretations. Most were very well done, however.

(8) I thought this report was excellent and it has certainly helped me this year in my teaching. The bulletins I have reproduced and have used in class with my elementary majors when we discussed certain topics.

(9) I suppose everyone would do this sort of thing a little differently, but such personality characteristics cannot be called improvements. I like all three volumes very much.

(10) I think the final report is excellent. It represents a staggering amount of effort.

(11) It is excellent!

(12) You are to be commended on the size and completeness of the report.

(13) If the bulletins were in book form, they'd be even more valuable for students as a single reference or supplementary text.

(14) I think you've missed a few. Let's keep a continuous up-date with new revisions every other year!

(15) Thank you for sending me your final report and the bulletins. They have made available information that has been interesting and useful but which I never would have searched out on my own.

(16) I've been too busy using this material to yet be concerned with improving it.

(17) The report is excellent! It's the research that is reported that needs some work! Thanks so much for the help you've given me through these volumes.

(18) The ideal thing would be to keep it updated. Also, a companion volume of other journals would be useful; also foreign work. Somehow we need to be able to cluster results into the beginnings of a theory.

(19) Perhaps an index of the many mathematical and related notions, relating concepts referred to in all three volumes, would be helpful to the reader. Perhaps even an author index of the studies cited would enhance the ability to readily use your reference books. They are very nicely put together. The summaries are excellent research guides. It would be an interesting project to periodically follow up on the research, in summary form, since the conclusion of your volumes.

(20) Evaluations of important projects, as SMSG, Madison, etc., in use in school systems would gather an important range of information in one place.

(21) Your bulletins are useful, particularly for classroom teachers. However, the reports are brief enough that the summary which precedes them gives an effect of redundancy. You could add more information if you cut down the summaries, using subtitles to organize the sections or capsules in the margins to highlight the report sections. [Phase II]

(22) Valuable contribution; good format; quick sharing of information; will be useful for some time. When provision is made for updating?

(23) Bully - but obviously dictated by economics. Wonder about updating. Would separate index - replaced as supplementary volumes of annotated bibliographies are produced - be useful in future? e.g.

1975 Index

A

.

.

B → Thesis
B → Final Report
B → 1972 Publication
B → 1975 Updating

.

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(Assuming the Suydamian task is a continuing one!)

B. Phase II Questionnaires

An Evaluation Questionnaire on Phase II (see Appendix F) was prepared for mailing in late May to those who had received the eleven bulletins. However, due to carelessness it was not put in the mail until the third week in June, so it reached few people who were leaving for summer addresses - and it didn't reach many until July, long after the requested return date. For these reasons, and because the mail service has been variable in delivery of the materials in various parts of the country, the responses were fewer than was hoped, even considering the fact that a mailed questionnaire can be expected to result in a small percentage of returns. By August 1, 469 questionnaires had been returned, and this number were analyzed. (More have been returned since, but have not been included.)

In general, these responses are favorable - so much so that one suspects a biased return. The number of each type of respondee is presented in Table III. The data on responses are summarized in Table IV, which gives both the number and the percentage responding to each question. Almost 74% indicated they found the bulletins "decidedly useful," about 25% indicated they were "somewhat useful," and only 0.2% checked "not at all useful," while 1% failed to answer. Bulletins A-4, "Individualizing Instruction" and B-5, "Verbal Problem Solving" seemed to be considered most helpful, while Bulletin A-6, "Planning for Research in Schools" was least helpful. However, it should be noted that the difference in numbers is slight.

The total number of users could not be determined with any degree of certainty (questions 13 and 14), but the use is varied as indicated by the response to question 15. Any other comments written on the form were evaluated: 98% of these were favorable.

The only categories with a large number of responses were those of college professor (101) and principal (104). Their responses were statistically analyzed, using a t-test; no significant difference in their responses was found.

Needless to say, the positive reactions are gratifying to view!

TABLE III

NUMBER OF PERSONS IN EACH CATEGORY
WHO RETURNED THE EVALUATION QUESTIONNAIRE FOR PHASE II

Respondee Categories	Number	Per cent
College teacher	101	21.5
College teacher of mathematics methods courses	89	19.0
College teacher of mathematics content courses	23	4.9
College teacher of other courses	0	0.0
Other position at college level	6	1.3
Principal	104	22.2
Principal of elementary school (k-6)	44	9.4
Principal of junior high school (7-9)	6	1.3
Principal of senior high school (10-12)	1	0.2
Classroom teacher	9	1.9
Classroom teacher of elementary school (k-6)	3	0.6
Classroom teacher of junior high school (7-9)	2	0.4
Classroom teacher of senior high school (10-12)	3	0.6
Mathematics coordinator/supervisor	28	6.0
Curriculum specialist	20	4.3
Student	7	1.5
Undergraduate student	0	0.0
Graduate student	3	0.6
Other	14	3.0

TABLE IV

SUMMARY OF DATA ON EVALUATION QUESTIONNAIRE FOR BULLETINS, PHASE II

	Qu. 2: Useful				Qu. 3: Helpfulness, Set A							
					A-1		A-2		A-3		A-4	
	Decidedly	Somewhat	No	Omit	Most	Least	Most	Least	Most	Least	Most	Least
Number	346	117	1	5	130	108	118	102	145	54	194	51
Per cent	73.8	24.9	0.2	1.1	27.7	23.0	25.2	21.7	30.9	11.5	41.4	10.9

	Qu. 3 (Continued)					Qu. 4: Helpfulness, Set B							
	A-5		A-6		All	B-1		B-2		B-3		B-4	
	Most	Least	Most	Least	Most	Most	Least	Most	Least	Most	Least	Most	Least
Number	107	104	47	229	43	104	169	140	108	152	65	103	160
Per cent	22.8	22.2	10.0	48.8	9.2	22.2	36.0	29.9	23.0	32.4	13.9	22.0	34.1

TABLE IV (CONTINUED)

	Qu. 4 (Continued) B-5 Most	Least	All Most	Qu. 5: Type Pedagogy	Theory	Omit	Qu. 6: Reading Clear	Ambiguous	Omit	Qu. 7: Detail Adequate	More Than Adequate
Number	188	98	51	353	81	35	454	10	5	390	41
Per cent	40.1	20.9	10.8	75.3	17.3	7.5	96.8	2.1	1.1	83.2	8.7

	Qu. 7 (Continued) Less Than Adequate	Omit	Qu. 8: Confidence Level Yes	No	Uncertain	Omit	Qu. 9: Validity Yes	No	Uncertain	Omit
Number	31	7	425	4	32	8	399	4	57	9
Per cent	6.6	1.5	90.6	0.9	6.8	1.7	85.1	0.9	12.2	1.9

TABLE IV (CONTINUED)

	Qu. 10: Bias			Qu. 11: Appropriateness			Qu. 12: Readers				
	Yes	No	Omit	Pre-service	Graduate Student	In-service	Myself	Others	In-service	Pre-service	Graduate Student
Number	8	437	24	301	322	414	143	273	223	97	94
Per cent	1.7	93.2	5.1	64.2	68.7	88.3	30.5	58.2	47.5	20.7	20.0

	Qu. 12 (Continued)		Qu. 15: Use					Qu. 16: Read	
	Admin./ Supv.	Other College	Pre-service	In-service	Disc./Study Groups	Curric. Comm.	Other	Entire	Only Overview
Number	72	68	109	162	174	110	100	420	26
Per cent	15.2	14.5	23.2	34.5	37.1	23.5	21.3	89.6	5.5

TABLE IV (CONTINUED)

	Qu. 16 (Continued)		Qu. 17: Copied				Qu. 18: Availability		
	Only Closer View	Omit	Entire	Only Overview	Only Closer View	Omit	Don't Reprint	Reprint	Separately
Number	2	21	327	95	39	8	36	396	97
Per cent	0.4	4.5	69.7	20.3	8.3	1.7	7.7	84.4	20.7

	Qu. 18 (Continued)		Comments	
	As Collection	Omit	Favorable	Unfavorable
Number	275	37	108	2
Per cent	58.6	7.9	23.0	0.4

V. SUMMARY

A. Conclusions

The reactions from those receiving the materials produced in the Interpretive Study give an indication that, for some sizeable number of educators, a need has been at least partially met. There is a continuing need to provide a readable, readily available source of information on the research on elementary school mathematics.

B. Recommendations

Many things have been learned during the course of this project. Some have been learned through mistakes, others through testing varied procedures until the one most efficient is found. Still other suggestions have been made by those receiving the materials, as noted previously. And there are recommendations for future directions for this project - if the U.S. Office of Education were to continue funding beyond this initial stage during which a base has been built.

Among the many recommendations which could be made are these, which have been considered for their applicability to other projects as well as this one:

(1) The details involved in printing and mailing materials such as the bulletins produced for this study should be carefully checked - and rechecked intermittently. The "headaches" resulting from printers who fail to meet deadlines or from mailing services who incorrectly process materials result in many sleepless nights.

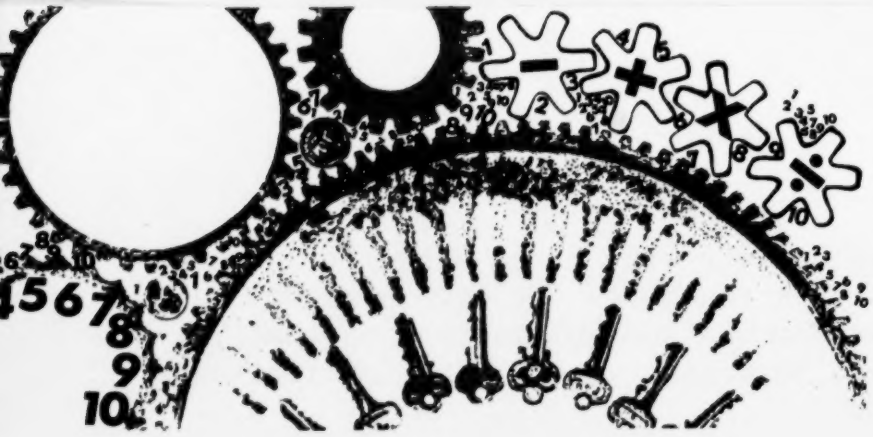
(2) The scope as well of the content of bulletins should be carefully determined prior to writing: writing to specifications facilitates this stage of the "production." When possible, the budget should allow for materials of sufficient length to allow for careful development and inclusion of all valid content. Many of the bulletins prepared for this study could have been longer, to include more studies and more details about these studies. Had space permitted, more illustrative materials could have been included.

(3) Projects which are funded as a dissemination effort should be considered for continuation of funds to provide continued service. It is somewhat pointless to develop an audience who indicate interest in receiving materials, and furthermore indicate that these materials are useful and helpful, only to terminate the effort because priorities have

shifted. There is a need to provide increasingly specific help, to answer a wider range of questions, to update the information regularly. It is to be hoped that other projects which have been successful at reaching proposed goals will find that they are allowed to do more than merely lay a foundation. Short-term efforts will not suffice to meet the need for synthesizing, interpreting, and applying research.

APPENDIX A

BULLETINS



Using Research: A Key to Elementary School Mathematics

Have you ever been asked . . .

What is the best mathematics program for all elementary schools?

What is the best way to teach mathematics to all elementary school pupils?

For such questions, it truly is a case of ANSWER: IMPOSSIBLE.

But there are questions about elementary school mathematics for which answers may be suggested -- answers which to one degree or another are based upon research findings.

PRINCIPALS:

Do you and your teachers seek guidance from research to improve your instructional programs in mathematics?

COLLEGE PROFESSORS:

Do you need a source of information and synthesis of research on elementary school mathematics?

The need for materials which communicate research findings on elementary school mathematics for use in the classroom is felt at both in-service and pre-service levels. Meeting this need is the intent of a project sponsored at The Pennsylvania State University by the U.S. Office of Education.* You'll find enclosed the first of a series of bulletins being prepared as part of this project, so that you can study an example of the type of materials to be provided.

These materials are being developed in two sets, A and B:

Set A. Each bulletin in this set will interpret selected research findings which pertain to a broad aspect of elementary school mathematics: attitudes toward mathematics, the teaching-learning process, planning for instruction, individualizing instruction, instructional materials and media, and planning for research.

Set B. Each bulletin in this set will focus on selected research findings which pertain to some aspect of the content of elementary school mathematics: addition and subtraction, multiplication and division, fractions and decimals, problem solving and related abilities, and other topics.

The first page of the bulletin on each topic will be on a tear-off sheet which could be distributed separately to teachers. It will present highlights and summaries, with illustrations and suggestions so that teachers may readily sense the extent to which research has implications for the classroom.

The pages which follow the tear-off sheet explore the topic in greater depth. Specific research is cited, with further analysis and comments for those involved in the preparation of teachers or in curriculum planning. Lists of selected references are included for further study. Both the tear-off sheet and the bulletin suggest answers to questions frequently asked about classroom practices.

If you would like to receive the other bulletins in both series, which will be mailed periodically, please write your name and address on the attached form. After folding it, close it with a staple, and mail it to:

Project on Interpreting Mathematics Education Research
Center for Cooperative Research with Schools
302 Education Building
The Pennsylvania State University
University Park, Pennsylvania 16802

By the way, the materials are free -- just indicate your interest by signing your name! You may reproduce any of the materials to meet your local needs.

Please contact the Project Director if you have any questions about the materials in general, or about any bulletin in particular. We also welcome your suggestions regarding topics you would like to see considered in the bulletins and things we might do to improve the usefulness of the bulletins.

---

* The project is sponsored by the Research Utilization Branch, Bureau of Research, U.S. Office of Education, under Grant No. OEG-0-9-480586-1352(010). It is entitled "Interpretive Study of Research and Development in Elementary School Mathematics," Marilyn N. Suydam, Project Director. The Project Consultant is J. Fred Weaver of the University of Wisconsin-Madison.

Yes, I'd like to receive the other bulletins interpreting research on elementary school mathematics.

Name _____

Address _____

_____ Zip Code _____

NOTE TO PRINCIPALS:

Your name was suggested as a key person to contact. We hope you will be able to suggest names of other principals who might be interested in receiving the materials. Please help us by indicating their names -- and continue the list on another sheet of paper if necessary!

NOTE TO COLLEGE PROFESSORS:

The National Council of Teachers of Mathematics indicated your name is listed in their files as a teacher educator -- but this doesn't include everyone. Please help us by listing the names of other mathematics educators in your college -- or elsewhere -- who didn't receive this folder.

Name _____

Address _____

_____ Zip Code _____

Name _____

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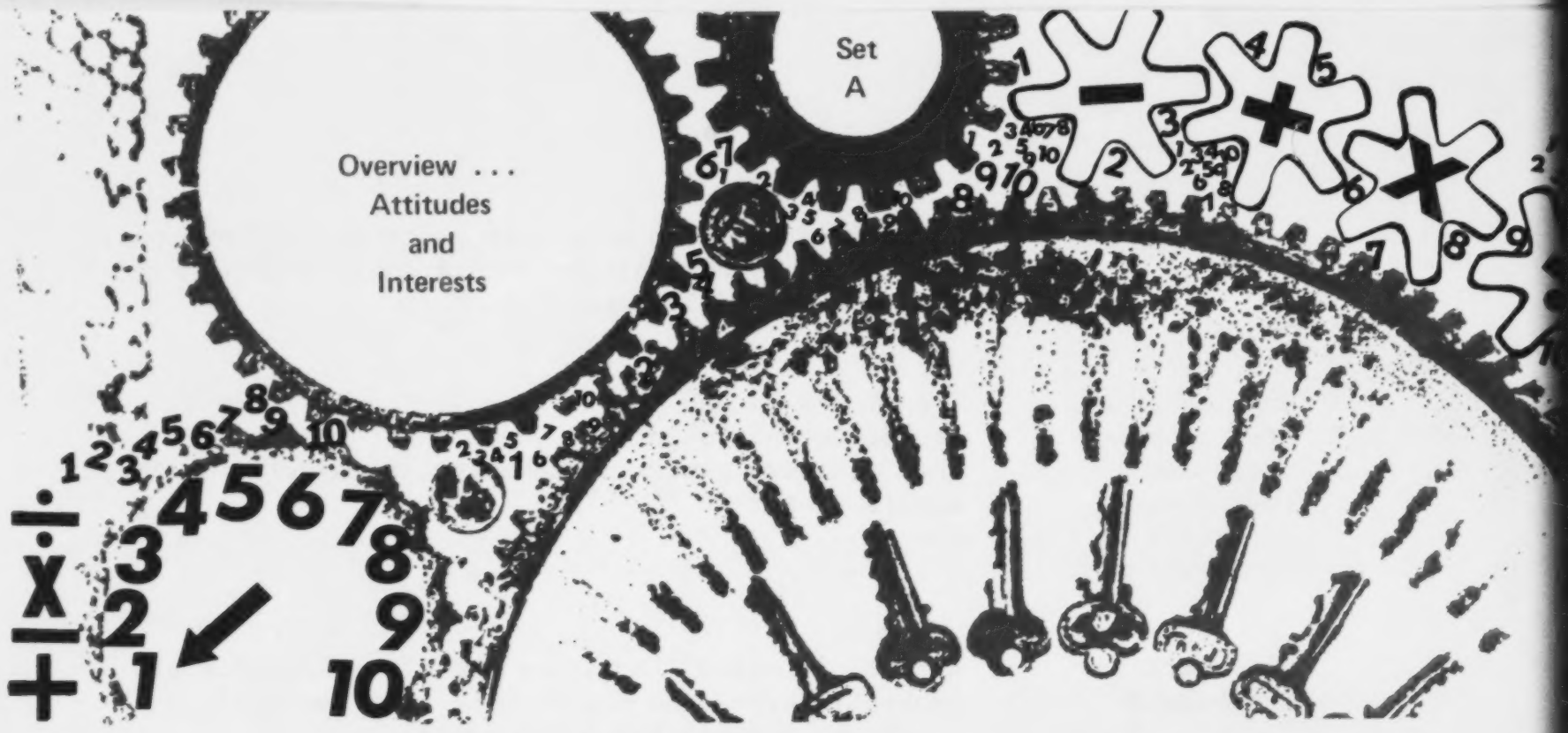


ATTITUDES AND INTERESTS

Do elementary school pupils like mathematics?

Many people believe that mathematics is disliked by most pupils -- or that it is just about the least favorite subject in the elementary school. It is true that in some surveys a significant proportion of pupils rated mathematics as the least liked of their school subjects. But it is equally true that in these surveys approximately the same proportion of pupils (at least 20%) cited mathematics as the best liked or the second best liked school subject.

Boys seem to prefer mathematics slightly more than do girls, especially toward the upper elementary grades.

Generally it has been found that pupils who like mathematics do so regardless of whether their program is contemporary or traditional. There also is evidence to show that fewer pupils are afraid of mathematics and more enjoy the challenge of mathematics problems today than pupils tended to ten years ago.

How important are attitudes and interests in mathematics?

First of all, there is no consistent body of research evidence to support the popular belief that there is a significant positive relationship between pupil attitudes toward mathematics and pupil achievement in mathematics. We have little research basis for believing that these two things are causally related.

Those studies which have been reported indicate only a trend or a low positive relationship between attitude and achievement.

How important are teachers' attitudes toward and interests in mathematics?

Research also has little to contribute by way of an answer to this question. Several studies show that when teachers prefer mathematics, a majority of their pupils prefer it.

What affects attitudes?

Attitudes toward elementary school mathematics are probably formed and modified by many forces:

(1) by parents and other adults
(2) by classmates and other children
(3) by teachers -- and the way they teach
(4) by the nature and demands of the subject itself
(5) by the learning style of the child.

Stressing the out-of-school usefulness of mathematics has been shown to help children to develop more positive attitudes toward it.

How can attitudes be improved?

Major reasons for pupils' dislike of mathematics include lack of understanding, high level of difficulty, poor achievement, and lack of interest in certain aspects of mathematics.

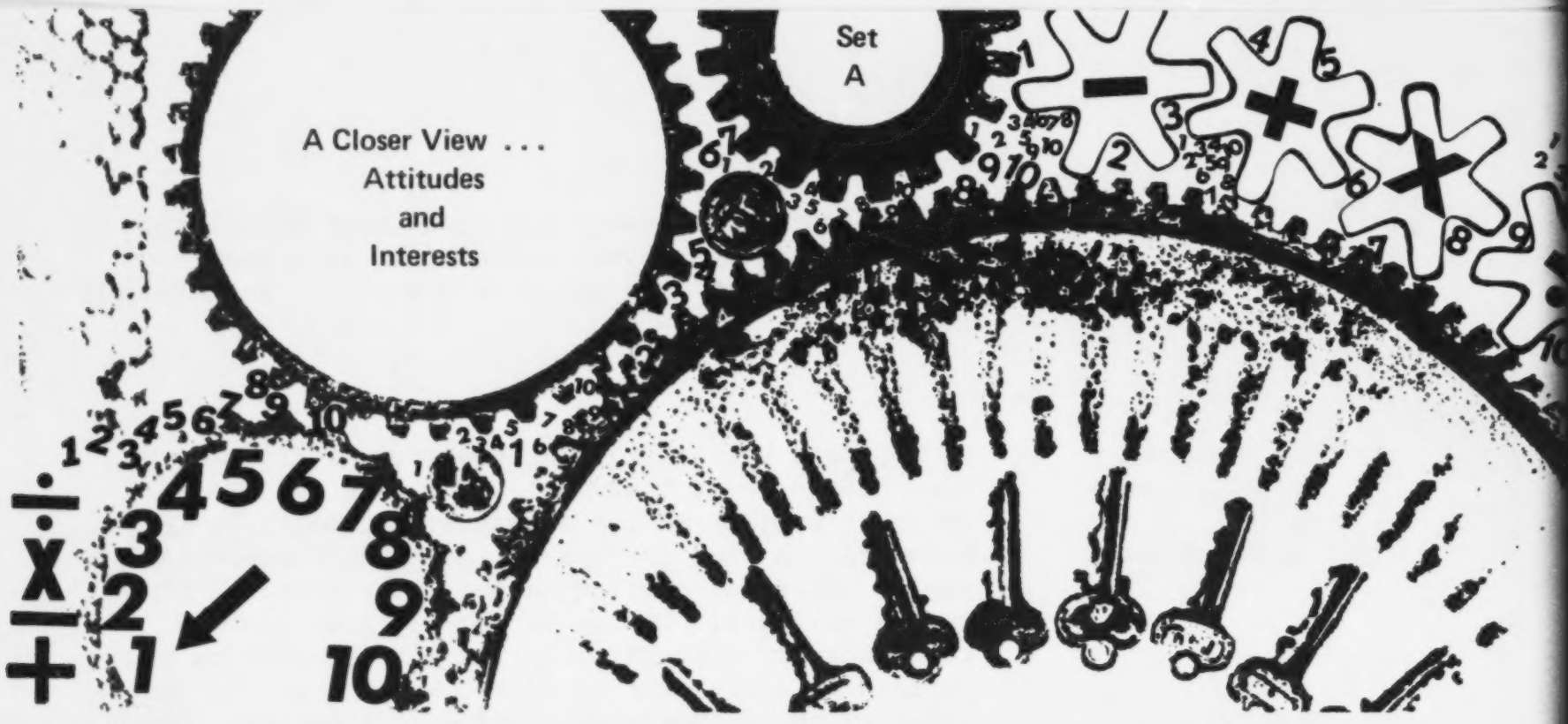
On the other hand, children like mathematics primarily because they find it interesting, challenging, and fun.

Many teachers have observed that interest and attitude can be improved if:

(1) realistic, short-term goals are established -- goals which pupils have a reasonable chance of attaining, and
(2) pupils are made aware of success and can sense progress toward these recognized goals.

The material included in this bulletin is a product of the "Interpretive Study of Research and Development in Elementary School Mathematics" (Grant No. OEG-0-9-480586-1352(010), sponsored by the Research Utilization Branch, Bureau of Research, U.S. Office of Education, and conducted at The Pennsylvania State University.

If you would like more information about the research whose findings are cited above, contact MARILYN N. SUYDAM, Project Director, at The Pennsylvania State University, University Park, Pennsylvania, 16802.



Using Research: A Key to Elementary School Mathematics

ATTITUDES AND INTERESTS

What are attitudes and interests?

Attitudes and interests are <u>affective</u> things, having to do with <u>feelings</u>. In this bulletin, we are concerned with how pupils and teachers feel about mathematics. Attitudes and interests are thought to exert a dynamic, directive influence on an individual's responses; thus attitudes and interests may be related to the teaching and learning of mathematics.

How are they sometimes investigated?

When are they formed by pupils?

Attitudes and interests frequently have been investigated by the use of scales on which agreement or the degree of agreement or disagreement with statements about mathematics is indicated. Sometimes various school subjects have been ranked by order of preference, or likes and dislikes have been indicated. Both methods obviously rely on the honesty of the individual in expressing his true feelings.

The material included in this bulletin is a product of the "Interpretive Study of Research and Development in Elementary School Mathematics" (Grant No. OEG-0-9-480586-1352(010), sponsored by the Research Utilization Branch, Bureau of Research, U.S. Office of Education.

The bulletin was prepared by MARILYN N. SUYDAM, The Pennsylvania State University, Project Director, and J. FRED WEAVER, The University of Wisconsin-Madison, Project Consultant. Art by Ed Saffell.

It should be noted that research is variable with respect to its quality; hence, the same degree of confidence cannot be placed in all findings. An attempt has been made to take this fact into consideration in preparing this bulletin.

The majority of evidence indicates that relatively definite attitudes about mathematics have been developed by the time children are in the intermediate grades.

Do elementary school pupils like mathematics?

Many people believe that mathematics is disliked by most pupils -- or that it is just about the least favorite subject in the elementary school. It is true that in some surveys a significant proportion of pupils rated mathematics as the least liked of their school subjects. But it is equally true that in these surveys approximately the same proportion of pupils (at least 20%) cited mathematics as the best liked or the second best liked school subject (Chase, 1949; Chase and Wilson, 1958; Curry, 1963; Faust, 1963; Greenblatt, 1962; Inskeep and Rowland, 1965; Mosher, 1952; Rowland and Inskeep, 1963; Sister Josephina, 1959).
Yes, individual differences do exist among pupils!

Dutton (1956, 1968) supported these findings with evidence from answers given on scales of items. Similarly, Stright (1960) reported that:

(1) 9% felt that mathematics was a waste of time
(2) 20% thought mathematics uninteresting
(3) 58% said it was the best subject in school
(4) 66% wished they had more mathematics
(5) 80% said they really enjoyed mathematics.

Boys seem to prefer mathematics slightly more than do girls, especially toward the upper elementary school grades (Chase and Wilson, 1958; Dutton, 1956; Stright, 1960).

How important are attitudes and interests in mathematics?

Does a more favorable attitude, or greater interest, lead to higher achievement?

First of all, there is no consistent body of research evidence to support the popular belief that there is a significant positive relationship between pupil attitudes toward mathematics and pupil achievement in mathematics. We have little research basis for believing that these two things are causally related.

Lyda and Morse (1963) reported that among fourth-grade pupils, significant gains in mathematics achievement were associated with a combination of meaningful instruction and an increase in the favorableness of attitude toward mathematics. Nothing could be asserted, however, about the relation between achievement and attitude per se.

Bassham, Murphy and Murphy (1964) observed "an important difference" in level of mathematics achievement between sixth-grade pupils who had relatively more favorable attitudes toward mathematics and those who had relatively less favorable attitudes. However, the investigators were not able to specify the level of confidence with which this finding could be accepted as a non-chance difference.

In investigations of the subject preferences of fifth-grade children, Chase (1949) reported no consistent pattern of relationship between pupils' relative preference for mathematics and their mathematics achievement level. Dean (1950), using some of the pupils involved in this study, found that pupils who did well in mathematics generally had indicated a preference toward it. However, preference for mathematics did not necessarily indicate that achievement would be better.

In a later investigation of pupils' subject preferences, Greenblatt (1962) reported a significant relationship between relative preference for mathematics and mathematical achievement level on the part of girls in grades 3-5, but no such significant relationship existed for boys.

Anttonen (1968) found consistent low correlations between attitude and achievement from fifth grade through high school. Faust (1963) and Shapiro (1962) also found a low positive relationship existed between attitude and achievement.

Intelligence, which cannot be separated from achievement, and its relationship to attitude was investigated by Rice (1963) and Greenblatt (1962), who noted that pupils with IQ's above 110 had a greater interest in mathematics.

How important are teachers' attitudes toward and interests in mathematics?

What is their relation to pupil attitudes and interests, and to pupil achievement?

Research also has little to contribute by way of answers to this set of questions which pertain to the influence of teachers' attitudes toward and interests in mathematics upon pupil attitudes, interests, and achievement.

Greenblatt (1962) reported a significant relationship between teacher preference for mathematics and pupil preference for mathematics in the case of children who had IQ's above 110. But no such significant relationship was found in the case of pupils in lower IQ groupings.

In the case of children in grades 4-6, Inskeep and Rowland (1965) found a non-significant correlation between teacher preference for mathematics and pupil preference for mathematics.

Chase (1949) reported a strong agreement between fifth-grade teachers' preference for mathematics and their respective pupils' preference for mathematics.

A decade later, in a replication of this investigation, Chase and Wilson (1958) reported no consequential change: when teachers preferred mathematics, a majority of their pupils preferred it.

Abrego (1966) found no relationship between achievement and attitude in either traditional or newer mathematics programs.

What is the attitude of pupils toward the new programs?

According to Hungerman's findings (1967), pupils' attitudes were similarly positive both for contemporary and conventional programs. But for each type of program there was a low positive relationship between IQ and attitude, and also between attitude and achievement.

More generally, pupils who like mathematics do so independently of the kind of program (contemporary or conventional).

Feldhake (1966) reported that high achievers found new mathematics programs more interesting than did low achievers.

Dutton (1968) observed that fewer pupils are afraid of mathematics and more enjoy the challenge of a mathematics problem today than pupils tended to ten years ago.

What affects attitudes?

Attitudes toward elementary school mathematics are probably formed and modified by many forces. The influence of other people could be named as one source: parents and other non-school-related adults, classmates and other children, and teachers in each of the grades.

The way in which the teacher teaches seems to be of importance -- the methods and materials he or she uses, as well as his or her manner, probably affect pupils' attitudes.

The subject itself undoubtedly has an influence on a child's attitude: the precision of mathematics when compared with many other subjects; the need for thorough learning of facts and algorithms; the "building block" characteristic wherein many topics are built and often dependent on previous knowledge. Indeed, mathematics has traditionally been considered difficult, and its use as a mental discipline tool is still unfortunately being touted and abused by some persons.

The learning style of the child is also an important factor to consider. The orderliness which discourages some is the very aspect which attracts others.

Studies by Dutton (1956, 1968), Lyda and Morse (1963), and others have indicated that for some children the practical value and usefulness of mathematics in out-of-class situations contribute to the development of more positive attitudes toward mathematics.

Stright (1960) reported that 95% of the over one thousand pupils she surveyed felt that mathematics would help them in their daily lives, while 86% classified mathematics as the most useful subject. Dutton (1968) noted, however, that fewer see the practical uses of mathematics now than ten years ago. Making pupils aware of the uses of mathematics seems related to developing more positive attitudes, yet newer programs have frequently tended to deemphasize this aspect.

How can attitudes be improved?

Dutton (1956) reported that major reasons for pupils' dislike of mathematics include lack of understanding, high level of difficulty, poor achievement, and lack of interest in certain aspects of mathematics.

On the other hand, children like mathematics primarily because they find it interesting, challenging, and fun.

We have good reason to believe that interest and attitude can be improved if:

(1) realistic, short-term goals are established -- goals which pupils have a reasonable chance of attaining, and
(2) pupils are made aware of success and can sense progress toward these recognized goals.

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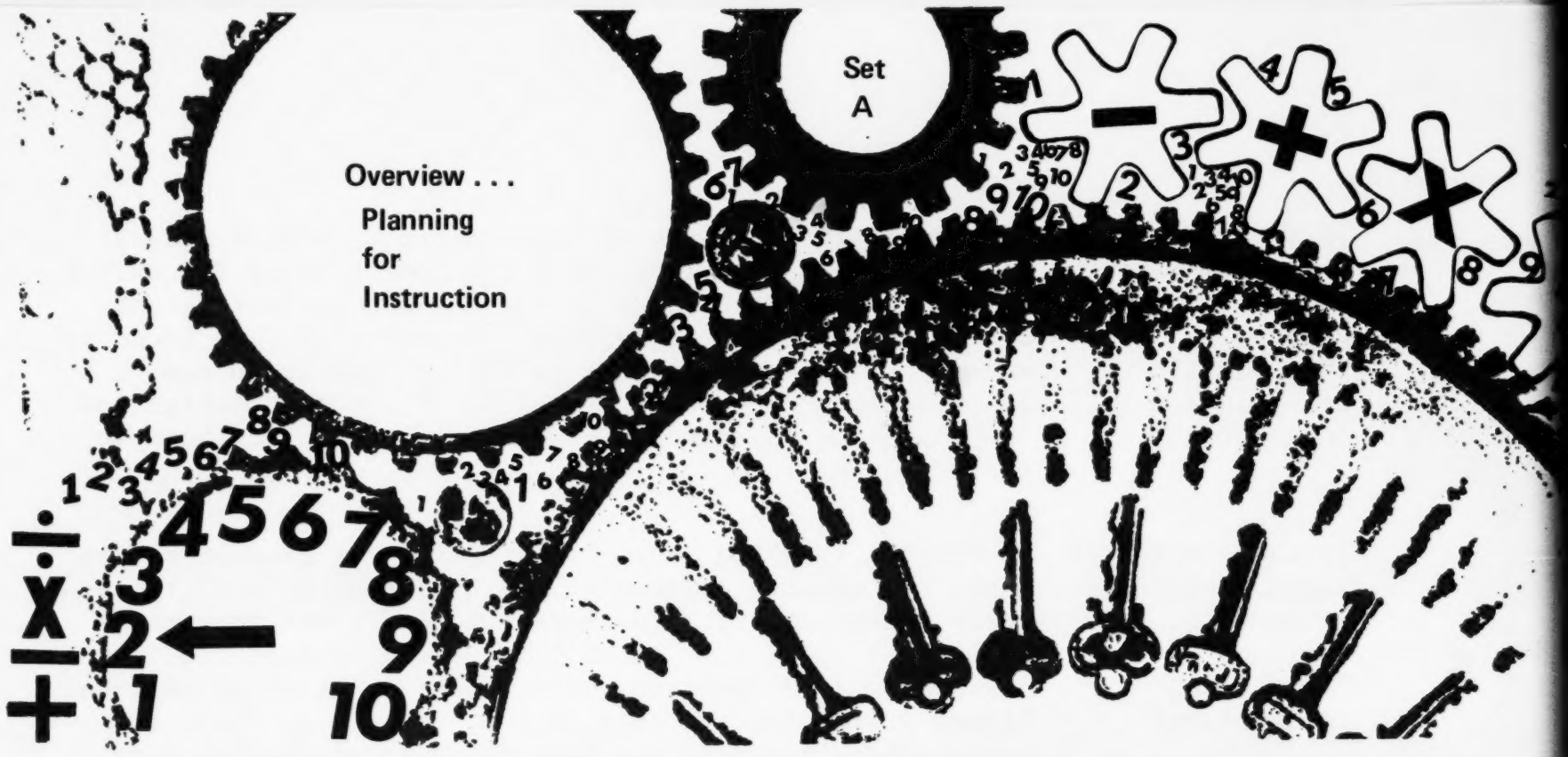

PLANNING FOR INSTRUCTION

Is there research to guide us in deciding . . .

. . . when to begin systematic instruction?	There is general agreement today that we will begin to teach mathematics systematically in grade 1, if not in kindergarten, since it has been shown that children can and do learn a great deal about number in the early years.
. . . how to organize the content for instruction?	Children can learn through an "activity method," if activities are (1) carefully planned to include sequential development of mathematical skills and (2) accompanied by strong drill programs. However, a program stressing sequential development with activities incorporated to introduce and reinforce concepts is generally advocated today. Emphasis in content organization is thus placed on the structure of mathematics, with consideration given to learning levels of children.
. . . how to organize schools for instruction?	No general conclusion can be drawn from research regarding the relative efficiency of any one organizational pattern for mathematical instruction. Neither team teaching nor departmentalization nor self-contained classrooms nor any other pattern appears to, per se, increase pupil achievement in mathematics. Perhaps the most important implication from various studies is that good teachers are effective regardless of the nature of classroom organization.

Is achievement in mathematics increased by a program of Individually Prescribed Instruction?	No substantial evidence to date supports an affirmative answer to this question. When the Individually Prescribed Instruction (IPI) program of the Oakleaf Project is considered, achievement of pupils has generally been found to be approximately equivalent to that of pupils in non-individualized programs. The type of research design and the measuring instruments used undoubtedly contribute to this finding.
Is there research which identifies outcomes of programs of "meaningful" instruction?	Meaningful teaching generally leads to (1) greater retention, (2) greater transfer, and (3) increased ability to solve independently. Teachers should (1) use more materials, (2) spend more class time on development and discussion, and (3) provide short, specific practice periods. Higher achievement in computation, problem solving, and mathematical concepts has been found to occur when more than half of the class time was spent on developmental activities, with the remainder on individual practice.
Is there research which identifies outcomes of "modern" or "contemporary" programs?	Generally, "modern" programs are as effective as "traditional" programs in developing "traditional" mathematical skills. Evaluation of groups taught with School Mathematics Study Group (SMSG) materials indicates that these groups can be expected to understand mathematical principles better than those using conventional materials. No significant differences in computational skills were reported, though results may vary depending on the type of test. It has been suggested that we can become so concerned with principles and properties that too little time is spent on computational practice and applications in social situations. Such practice and applications must be carefully planned. . . .

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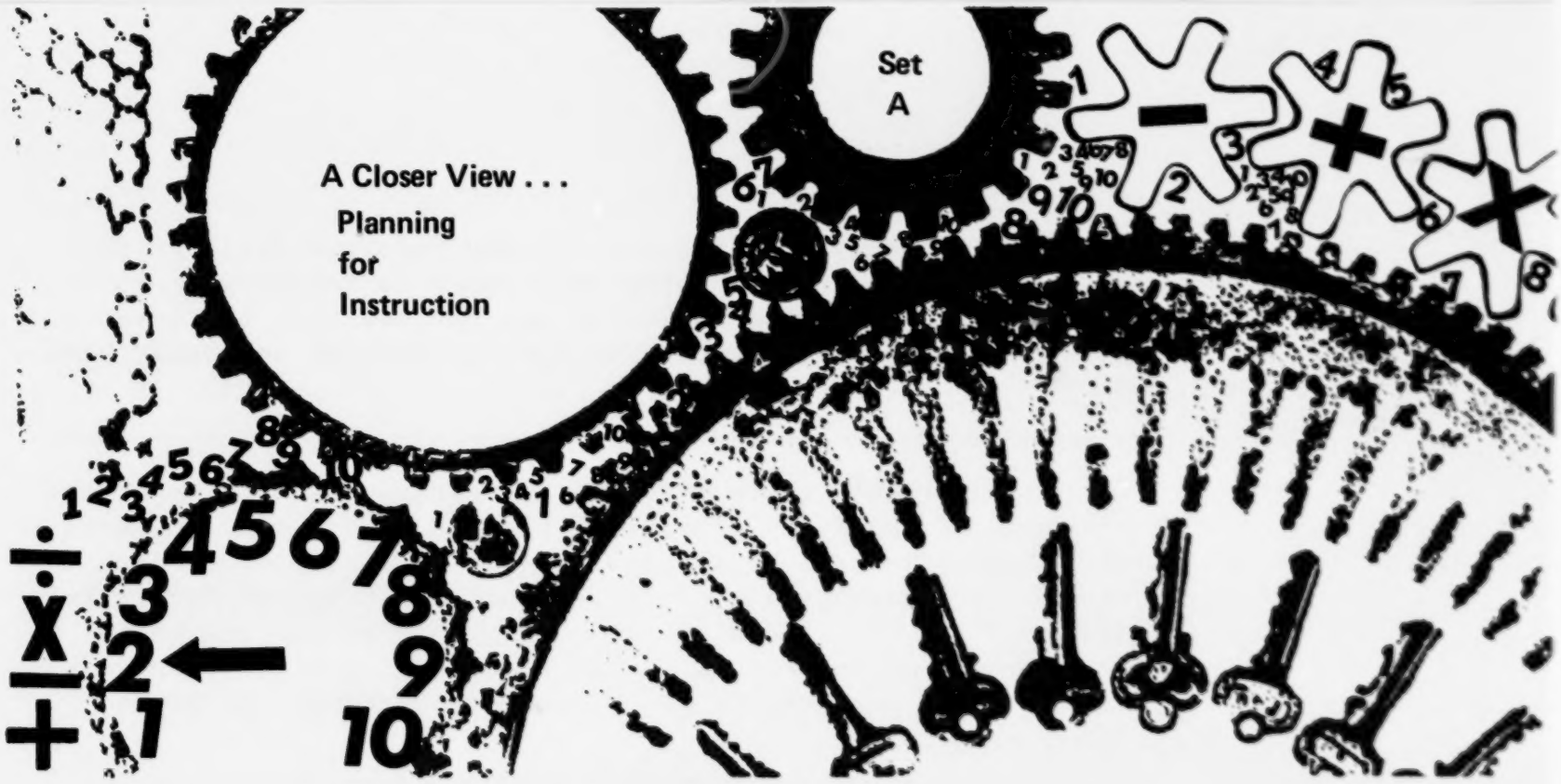

PLANNING FOR INSTRUCTION

Is there research to guide us in deciding . . .

. . . when to begin systematic instruction?

With a few exceptions, there is general agreement today that we will begin to teach mathematics systematically in grade 1, if not in kindergarten. Forty years ago, however, this was a matter of great debate. It was argued that formal study should be deferred "until the child could understand more and had a need for using mathematics." Therefore, until at least the third grade, mathematics should be learned "incidentally," through informal, unplanned contacts with number.

Opponents argued that such delay was a waste of time. Data to support this were collected; for instance, Washburne (1928) found that pupils who began mathematics in either grade 1 or 2 made better mathematics scores in grade 6 than did pupils who began mathematics in grade 3.

On the other hand, Sax and Ottina (1958) found more recently that by seventh grade, there was no significant difference in computation scores. Meaning scores were higher for pupils in a

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It should be noted that research is variable with respect to its quality; hence, the same degree of confidence cannot be placed in all findings. An attempt has been made to take this fact into consideration in preparing this bulletin.

school in which formal instruction was deferred until fifth grade. However, with the emphasis today on teaching an increased amount of mathematics at any earlier age, the question of when to begin systematic instruction has not seriously been reopened.

. . . how to organize the content for instruction?

During the 1930's there were many investigations of the effectiveness of "activity programs," planned to acquaint children with number as part of the environment. Generally research showed that mathematics could be learned through an "activity program," if (1) carefully planned to incorporate sequential development of mathematical skills and (2) accompanied by strong drill programs (e.g., Wilson, 1930; Harap and Mapes, 1934; Wrightstone, 1935).

For years the work of Washburne (1928) and the Committee of Seven strongly influenced the sequencing of topics in the curriculum. This group of superintendents and principals in the midwest surveyed pupils to find when topics were mastered, and then suggested the order and mental age or grade level in which each should be taught.

With the curriculum reform movement which began in the 1950's, much reorganization of content has been suggested. Generally, various topics and patterns have been "tried out" to see if they could be taught at a proposed level: research reflects many such trials. Gagne has long been working on the development of hierarchies of learning tasks. Suppes (1969) is approaching the problem of organization and sequencing with the aid of computer-stored data on pupils' responses.

. . . how to organize schools for instruction?

Educators have long searched for the "perfect" organizational pattern to meet individual pupil needs and increase achievement. A vast number of studies have been conducted to attempt to ascertain the efficacy or the superiority of departmentalization, team teaching, multi-graded, non-graded, or self-contained classrooms. However, attempts to isolate and measure the effects of any of these is extremely difficult, since factors such as content organization and teacher background interact with the pattern. The definitions of the various patterns also tend to overlap--what one person labels team teaching another defines as departmentalization, etc.

It is apparent from a review of the research that no general conclusion can be drawn regarding the relative efficiency of any one pattern for mathematics instruction. There appears to be no one pattern which, per se, will increase pupil achievement in mathematics. A proponent of any pattern can find studies that verify his stand. Achievement differences are affected more by other variables such as the mathematical background of the teacher, than by the organizational pattern. Perhaps the most important implication of the various studies

is that good teachers are effective regardless of the nature of classroom organization (Gibb and Matala, 1962).

Is achievement in mathematics increased by a program of Individually Prescribed Instruction?

No substantial research evidence has been reported to date to support an affirmative answer to this question. It refers to the project on Individually Prescribed Instruction (IPI) originated as a cooperative venture of the University of Pittsburgh's Learning Research and Development Center and Oakleaf Elementary School of the Baldwin-Whitehall School District of Pittsburgh.

In a recent Progress Report on IPI (1969) it was concluded that "on standard achievement tests IPI pupils do as well as non-IPI pupils." No claim is made for higher achievement on the part of IPI pupils. For instance, at the third, fourth, and fifth grade levels Fisher (1968) found no significant achievement differences under three instructional treatments: (1) IPI, (2) "programmed learning instruction," and (3) "standard classroom instruction."

In an inconclusive investigation of IPI effects among low, average, and high ability fourth, fifth, and sixth grade pupils, Deep (1967) questioned the appropriateness of standardized tests for measuring achievement within the IPI context. Other assessment problems and instructional factors associated with IPI have also been studied. Findings from such research and evaluation have been used to revise the program.

Is there research which identifies outcomes of programs of "meaningful" instruction?

[For "discovery-oriented instruction," see Bulletin A-3.]

Earlier in this century, it was doubted that children needed to understand what they learned. It was enough if they developed high degrees of skill. To take time to give explanations and develop understanding was deemed wasteful, besides being perplexing to the learners.

Then came the realization that certain things were to be gained if content made sense to the learner. When mathematics is taught according to the mathematical aim, learning becomes meaningful; when taught according to the social aim, significant. Children do not necessarily acquire meanings when they engage in social activities involving mathematics. Significant mathematical experiences need to be supplemented by meaningful mathematical experiences.

Dawson and Ruddell (1955) summarized studies, such as those by Swenson, Anderson, Howard, and Brownell and Moser, which were concerned with various aspects of meaning. They concluded that meaningful teaching generally leads to: (1) greater retention, (2) greater transfer, and (3) increased ability to solve independently. They also suggested that teachers should (1) use more materials, (2) spend more class time on development and discussion, and (3) provide short, specific practice periods.

Studies since that date have supported these findings. Greathouse (1966), for instance, found that groups taught by a

group-oriented meaningful method achieved more than those taught by individually-oriented meaningful methods, but each achieved more than a group taught by a drill-computation method. Miller (1957) found that "meaning" methods were more effective for most computational areas and for understanding of the principles of mathematics. The "rule" method, however, seemed more effective for low IQ children.

To determine how the use of class time affects achievement, Shipp and Deer (1960) compared four groups, in which 75%, 60%, 40% or 25% of class time was spent on group developmental work while the remainder was spent on individual practice. Higher achievement in computation, problem solving and mathematical concepts was obtained when more than half of the time was spent on developmental activities.

In replications of this experiment, Shuster and Pigge (1965) and Zahn (1966) used other time allocations. They confirmed the finding that when the greater proportion of time is spent on developmental activities, achievement is higher.

Hopkins (1966) compared two fifth grade groups which spent 50% time on meaningful activities and 50% time (1) on practice or (2) in informal investigations of more advanced concepts. No significant differences between computation scores for the two groups were found, but significant differences on understanding measures occurred. Hopkins concluded that the amount of time spent on practice "can be reduced substantially and still retain equivalent proficiency in arithmetic computation." If activities are carefully selected, understanding can be increased.

Is there research which identifies outcomes of "modern" or "contemporary" programs?

Payne (1965) summarized several studies and reported that "modern" programs were as effective as "traditional" programs in developing "traditional" mathematical skills; this is supported by more recent studies. There is evidence that "modern" materials are appropriate for a wide range of student abilities.

One phase of the National Longitudinal Study of Mathematical Abilities (NLSMA) compared achievement patterns based on 38 measures over a three-year period, for programs in grades 4-6 represented by six textbook series--three "modern" and three "conventional" (Carry and Weaver, 1969). The conjecture that achievement patterns would be more similar within textbook classifications ("modern" and "conventional") than across classifications was not supported consistently by actual findings of the investigation. It was not uncommon to identify subtests on which large differences in achievement existed among the "modern" textbook groups and also among the "conventional" textbook groups. Furthermore, the findings did not agree consistently with the hypothesis that "modern" and "conventional" texts could be distinguished on the basis of achievement level associated with particular subtests--although there was a trend in support of this conjecture.

Many persons feel that the School Mathematics Study Group (SMSG) has had the greatest impact on the curriculum of any experimental program. Certainly much research and evaluation has been concerned with the SMSG materials.

Hungerman (1967) and Grafft and Ruddell (1968) compared sixth grade classes who had studied the SMSG program during grades 4, 5, and 6, with classes who had studied a conventional program. Grafft and Ruddell reported that the SMSG group understood principles of multiplication better than did the conventionally taught groups, while no significant differences in computation were found. Hungerman found that achievement data significantly favored non-SMSG groups on a test of conventional arithmetic, and the SMSG group on a test of contemporary mathematics. Several other researchers reached this same conclusion in studying other "modern" programs.

Sloan and Pate (1966) studied teaching strategies, reporting that more SMSG teachers than teachers of "traditional" mathematics used analysis and comprehension questions, eliciting spontaneous responses, and developing content. The non-SMSG teachers used recall and recognition questions to a greater degree than any of the other questions they might have used.

One caution is included in several reports: we can become so concerned with principles and properties that little or no opportunity is given pupils to practice computation or apply mathematics in social contexts. Such practice and applications must be planned for. . . .

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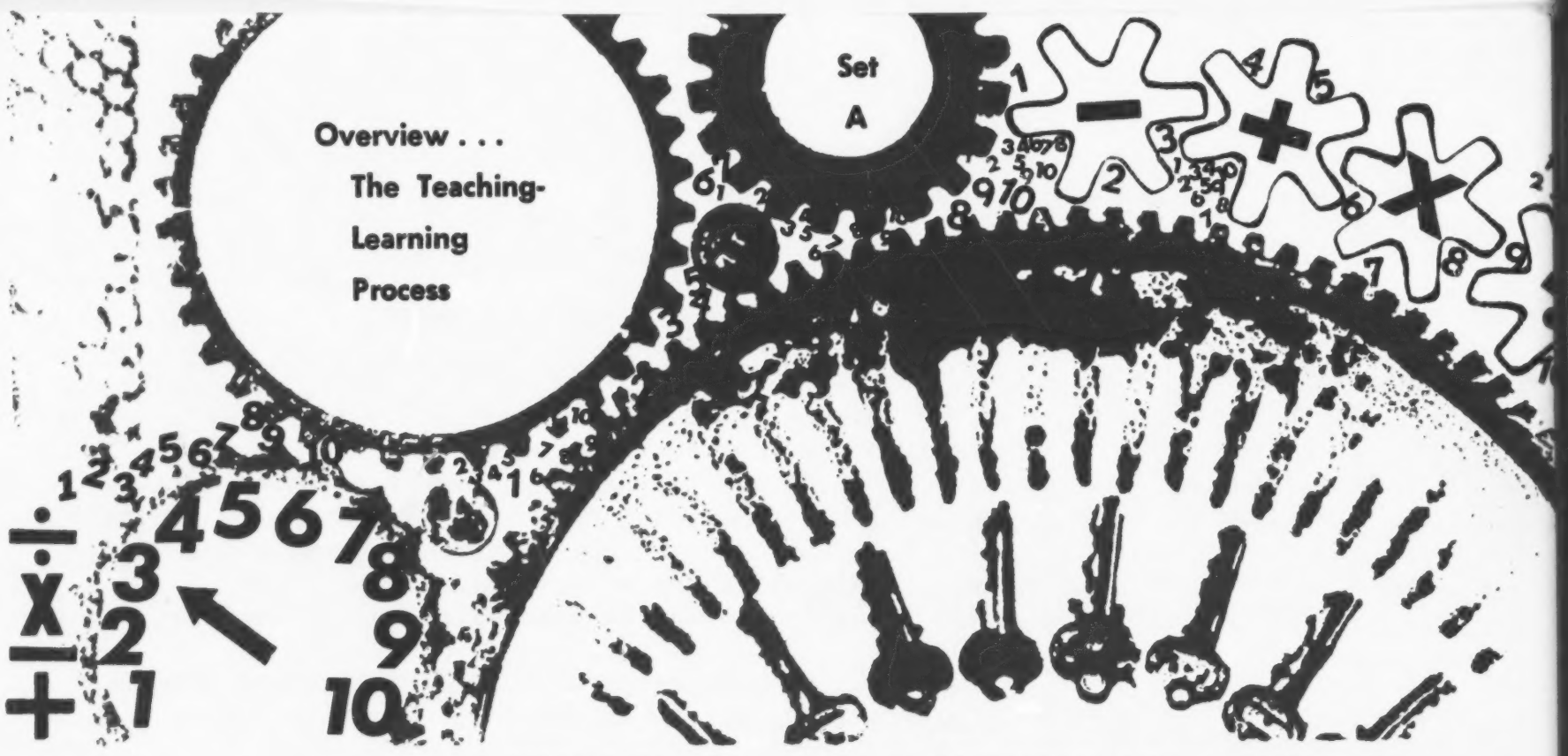

THE TEACHING-LEARNING PROCESS

What factors associated with the learner influence achievement in mathematics?

Children appear to acquire mastery and understanding of mathematical ideas in steps or stages. Materials which are carefully structured to guide children through various "levels" tend to promote retention of the knowledge.

Age and intelligence are positively correlated with ability to learn various concepts, and thus must be considered in your planning.

Is there research to guide us in motivating learning?

It seems plausible that children must be interested in learning in order to learn. What promotes interest? Games and materials are effective; your enthusiasm and praise of their efforts are <u>essential</u>. For some children, material "rewards" may be helpful.

Is there research to guide us in reinforcing learning?

Giving children "knowledge of results," by providing scores or correct answers, seems to be one of the best ways of reinforcing their learning. Confirming a child's response is more effective than merely supplying him with the answer.

Is there research to guide us in facilitating retention?

When an experience has meaning to the learner and is understood by the learner, retention is facilitated. Planning to spend at least 50% of mathematics class time on meaningful developmental activities will help, as will allowing children to work at their own level.

Intensive and specific review and practice should be provided, regularly and systematically, with especially careful review of material taught just before a vacation period.

Is there research to guide us in facilitating transfer?

You can help children to transfer mathematical skills and concepts from one experience to another by:

(1) planning and teaching for transfer--which implies that what is to be transferred must first be carefully determined

(2) teaching children how to transfer--which includes stress on searching for patterns and rules

(3) guiding children to generalize on the basis of experiences

(4) teaching with meaning--possibly discovery-oriented

(5) providing for instruction and practice for each child on his own level.

What is the interaction of organization and instruction variables?

The way in which the curriculum is organized--whether by areas or topics--and the way instruction is presented--either inductively or deductively--were not found to interact significantly to affect mathematical learning.

What is the role of "discovery" in the teaching-learning process?

There is much discrepancy in the way in which "discovery" is defined and used. If it is applied to a teaching approach in which the teacher leads pupils to a desired conclusion or behavior with directed questions, then it may be labelled "guided discovery." This is frequently contrasted with an "expository" approach, in which teachers explain or tell pupils what they are to do to perform a desired behavior.

When a "guided discovery" and an "expository" approach are compared, "guided discovery" groups have generally been found to achieve higher on tests of (1) retention and (2) transfer. Those taught by an "expository" approach may achieve higher scores on tests immediately following instruction.

Generally, the "guided discovery" groups achieve higher scores for problem solving than do groups taught by "exposition." However, neither approach has an advantage on measures of computational skill.

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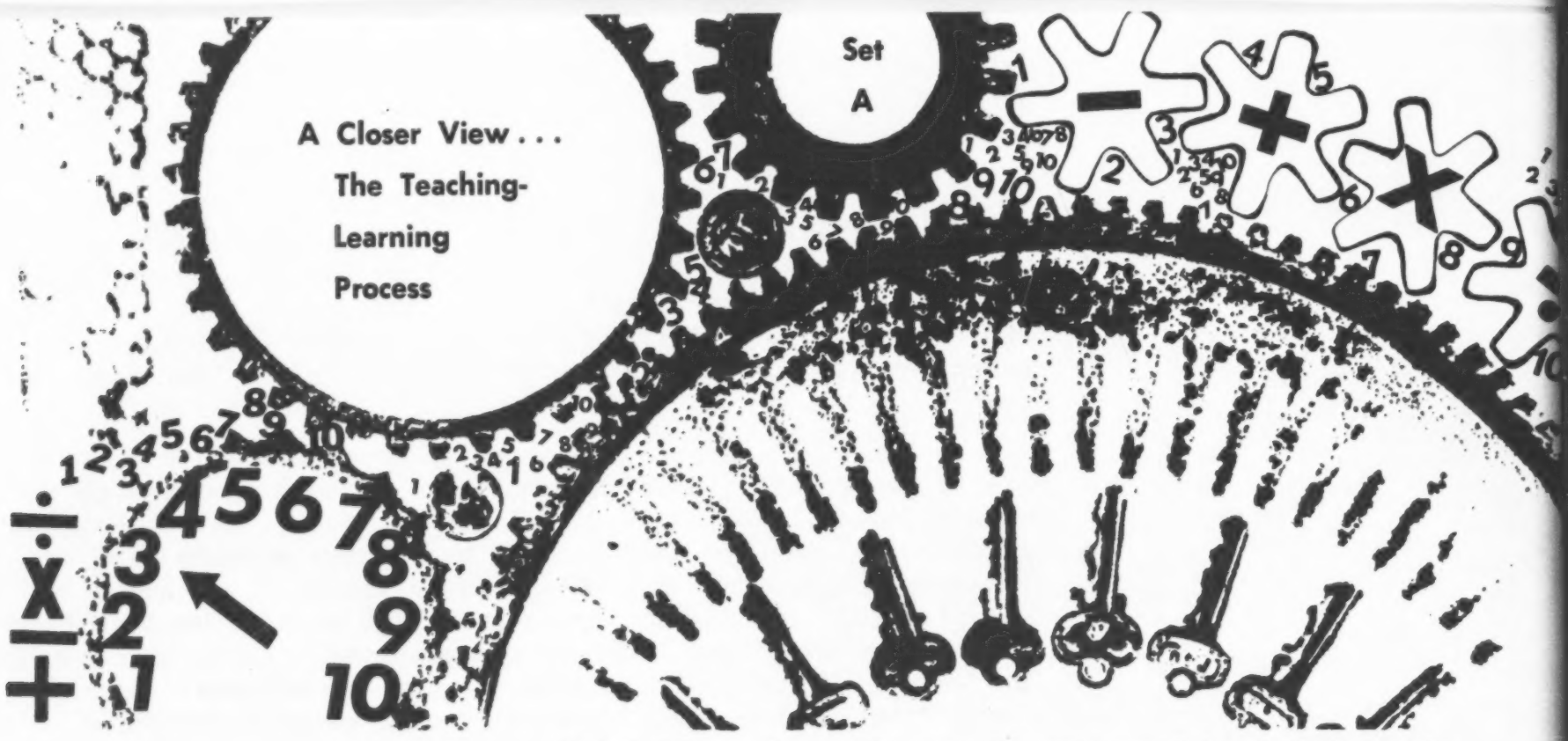

THE TEACHING-LEARNING PROCESS

Research to guide us in determining how we should teach and how children learn encompasses far more than one curriculum area. We have not attempted a broad survey of learning theory, but rather have selected that research which (1) is based on a phase of the elementary school mathematics curriculum and (2) provides specific suggestions to teachers of elementary school mathematics. Many of these findings have been substantiated not only in research across many phases of the curriculum, but also by practical use.

What factors associated with the learner influence achievement in mathematics?

Learning is not an "all or none" process. We generally acquire understanding progressively, in steps or stages. Perreault (1957) reported that the child's ability to count, to group, and to perceive the number of objects without counting appeared to reflect such developmental stages.

Gagne and Bassler (1963) structured a hierarchy of "subordinate knowledge" which led to the development of a concept. They found that, in general, sixth grade pupils learned concepts developed according to such a hierarchy. Although they did not retain all of the subordinate knowledge, they did continue to achieve well on the final task.

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It should be noted that research is variable with respect to its quality; hence, the same degree of confidence cannot be placed in all findings. An attempt has been made to take this fact into consideration in preparing this bulletin.

Brownell (1944) supplied interview data to support the conception of learning as a series of progressive reorganizations of processes and procedures. Hill (1961) found that children aged 6 through 8 could recognize the validity of logical inferences, with a pattern of steady growth rather than fixed stages.

Much additional research has shown that age and intelligence are highly related to ability to learn various specific mathematical ideas. Westbrook (1966), for instance, noted that the intellectual factors of reasoning and verbal meaning were related to achievement in mathematics in grades 4, 5, and 6. Meconi (1967) found that pupils with high ability were able to learn under any method that he investigated. Large variations in generalization ability, depending on the mathematical concept, intelligence level, and the visual pattern presented, were found on tests of varied mathematical content (Ebert, 1946).

Cathcart and Liedtke (1969) suggested that pupils in grades 2 and 3 who were identified as having a "reflective" learning style took longer to consider their responses and achieved better than pupils with an "impulsive" style. Certainly learning style needs to be considered as we plan lessons and give directions.

Is there research to guide us in motivating learning?

Exactly what "motivation" is has been the subject of some debate. Let us assume that it includes what the teacher does to increase pupils' interest in learning mathematics. (We further hope that increased interest will lead to increased achievement.) There are numerous reports about various games and materials which teachers have used successfully in increasing interest. The effect of teacher enthusiasm cannot be taken lightly.

What the teacher says--and how he says it--has been found to be particularly important. Not surprisingly, praise has been found to be a highly effective way to motivate.

Hollander (1968) recently studied the effect of different types of incentive on inner-city fifth and sixth graders following a test on addition and subtraction problems. He found that pupils worked faster when told they could earn a candy bar if they improved their own scores on a second test, and with greater accuracy when told they had performed exceptionally well. Those reproved by being told their scores were very low attempted fewer items and made more errors than were made under any of the other conditions.

Is there research to guide us in reinforcing learning?

One of the best ways of reinforcing learning is to give the child "knowledge of results"--by providing scores or by providing correct answers. Paige (1966) found that immediate reinforcement after a testing situation resulted in significantly

higher achievement scores later. Having the student respond and then giving confirmation is more effective than prompting him with the correct answer before giving him a chance to respond (McNeil, 1965.)

Kapos, Mech and Fox (1957) studied the effect of various amounts and patterns of reinforcement with third and fourth graders at several IQ levels. Different patterns of reinforcement produced differences in achievement. However, there was no clear indication of which quantity or pattern of reinforcement was best, nor was any relationship with IQ found.

Is there research to guide us in facilitating retention?

Obviously, we want children to retain what we are teaching and they are learning. There is much research to show that when something has meaning to the learner and is understood by the learner, he will be more likely to remember. Furthermore, Shuster and Pigge (1965) state that retention is better when at least 50 per cent of class time is spent on meaningful, developmental activities. Klausmeier and Check (1962) reported that when a pupil solved problems at his own level of difficulty, retention was good regardless of IQ level.

Burns (1960) reported that intensive, specific review will facilitate retention. He prepared lessons which included not only practice exercises, but also review study questions which directed pupils' attention to relevant things to consider. Meddleton (1956) pointed out that such review should be systematic.

Many teachers have noted that children fail to retain well over the summer vacation. The amount of loss varies with the child's ability and age, but how long before the vacation material was presented is especially important. Practice during the summer and review concentrated on materials presented in the spring have been shown to be especially helpful. Scott (1967) reported no systematic relationship of amount of loss and type of program, whether "traditional" or "modern."

Is there research to guide us in facilitating transfer?

Transfer infers that something learned from one experience can be applied to another experience. For instance, Olander (1931) found that pupils who studied 110 addition and subtraction combinations could give correct answers to the 90 untaught combinations. What facilitated this transfer best was instruction in generalizing, in teaching children to see patterns. Transfer increases as the similarity of problems and experiences increases. Much research has shown that meaningful instruction aids in transfer of learning. Recent studies also show that transfer is facilitated by discovery-oriented instruction.

In most studies is the implication that transfer is facilitated when teachers plan and teach for transfer--and we must teach children how to transfer. Kolb (1967), for instance,

carefully planned to have children transfer mathematical instruction to quantitative science behaviors, and achieved this transfer.

In general, the older the child and the higher his ability level, the better he can transfer. However, Klausmeier and Check (1962) found that children of various IQ levels transfer problem solving skills to new situations when the children were given work at their own level of difficulty.

What is the interaction of organization and instruction variables?

Armstrong (1968) studied the relative effects of two forms of spiral organization (area or topical) and two instructional modes of presentation (inductive or deductive). Sixth graders were assessed at each of six cognitive levels, within three areas (set theory, number theory, and geometry) and on four topics (terminology, relations, operations, and properties). The inductive mode of presentation fostered the learning of operations, while the deductive mode resulted in greater learning of mathematical properties. The interaction of curriculum organization and instructional presentation variables was not found to significantly affect mathematical learning.

What is the role of "discovery" in the teaching-learning process?

Few teachers are unaware of the word "discovery"--but there is much discrepancy about what it means as well as how it can be used. Research evidence is equivocal; perhaps the greatest factor contributing to this is the labelling of quite different methods with the same name. Nevertheless, findings from research on discovery have particular implications as we plan for the developmental aspects of mathematical teaching and learning.

In a pilot study with a small group of ten second-graders, Bassler (1968) provided groups with "intermediate guidance" in which pupils were led to a desired behavior through a "guided discovery" approach with directed questions by the teacher, or with "maximal guidance" in which teachers specifically told students what they were to do, followed by practice. The pattern of differences for posttest and retention achievement favored the "intermediate guidance" group. This group had higher transfer scores immediately following instruction, while the "maximal guidance" group had higher transfer scores on the retention test.

Fleckman (1967) reported that classes of fifth and sixth graders taught division by a "guided-discovery" method learned more concepts than classes taught by conventional textbook procedures, while computation was equivalent.

Scandura (1964) conducted several studies concerned with "exposition" versus "discovery" in classification tasks. He found that pupils taught by "discovery" were (1) better able to handle problem tasks, (2) took longer to reach the desired level of facility, and (3) seemed more self-reliant.

In an excellent study with fifth and sixth graders, Worthen (1968) compared two methods that differed in terms of sequence characteristics. In the expository method, the verbalization of the required concept or generalization was the initial step in the sequence. Mathematical principles were explained verbally and symbolically to the pupil, who then worked with examples. In the discovery method, the pupil was presented with an ordered, structured series of examples of a generalization. No explanation was given, nor any hint that there was an underlying principle to be discovered. The pupil was expected to acquire the mathematical concept or generalization through an inference of his own.

The two sequences of presentation, with carefully described teaching behaviors, resulted in significantly different pupil performance on several types of tests. In general, Worthen's findings support many of the claims made by proponents of discovery methods. The expository method was better than the discovery method on the initial test of learning, but discovery was better on retention tests administered after five and eleven weeks.

The discovery group also transferred concepts more readily and used discovery problem solving approaches to new situations better. No differences were found in pupil attitude toward the two approaches. The results further indicate that the discovery method need not take more time.

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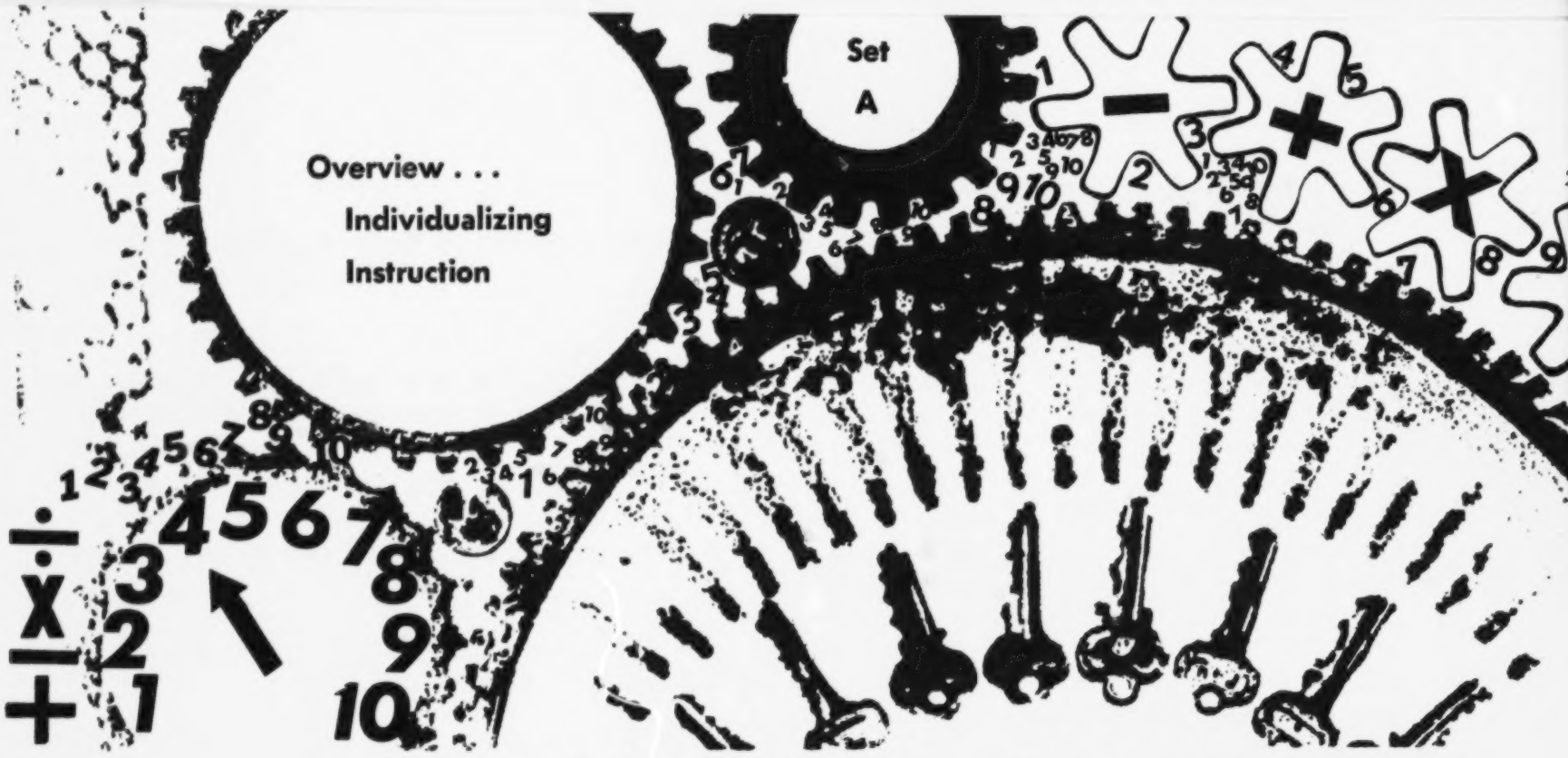

INDIVIDUALIZING INSTRUCTION

What factors are important to consider when individualizing instruction?

It seems apparent that there is no one best way to individualize instruction. You must identify various factors related to achievement and interest in mathematics, and then decide on appropriate variations in content, materials, method, and time.

Mathematical ability has been found to be a combination of intellectual, numerical, and spatial factors, with a verbal factor which is highly related to intelligence. It has been suggested that certain personality factors or emotional difficulties may be more important than intelligence as a factor contributing to lack of success in mathematics. Socioeconomic status also influences achievement, with achievement level increasing as socioeconomic level of the parent increases.

Some research has indicated that some students can be identified who will achieve better when taught inductively, while others learn better when taught deductively. Thus using a method appropriate to the learner is one way of individualizing instruction.

Should boys and girls have a different mathematics program?

While some researchers have reported that boys tended to score higher in mathematical reasoning and girls were better on fundamentals, most concluded that what little difference exists is not sufficient to influence curriculum decisions.

How does diagnosis aid in individualizing instruction?

You should ascertain the specific errors which a pupil is making, determine specifically how he works, and give specific remedial help.

Diagnostic tests for skills are available, and some tests which focus on understanding of mathematical ideas are available. You may find that observing and questioning children as they work is one of the best ways of ascertaining how they think as they do mathematics. These techniques provide you with information on what and how to teach him.

A testing-reteaching-retesting strategy will help to decrease the errors pupils make.

What types of grouping are effective?

Grouping on the basis of ability has been found in some studies to be especially effective for those at upper ability levels. The findings of research on grouping on the basis of achievement have been much more variable. Apparently the most important factor in grouping is the teacher: a good teacher will be successful regardless of the pattern of grouping used.

What is the effect of acceleration?

In general, acceleration has been reported to be effective for some children. Unfavorable academic, social, emotional and physical problems seem to be minimal when children are carefully selected and the program is carefully planned.

How may instruction be effectively individualized?

All in all, there is little substantial evidence to date indicating that programs of individualized mathematics instruction will lead to higher levels of pupil achievement when compared with non-individualized programs. Perhaps how each teacher teaches is the most significant factor, and obscures differences between the two types of programs.

The material included in this bulletin is a product of the "Interpretive Study of Research and Development in Elementary School Mathematics" (Grant No. OEG-0-9-480586-1352(010), sponsored by the Research Utilization Branch, Bureau of Research, U.S. Office of Education, and conducted at The Pennsylvania State University.

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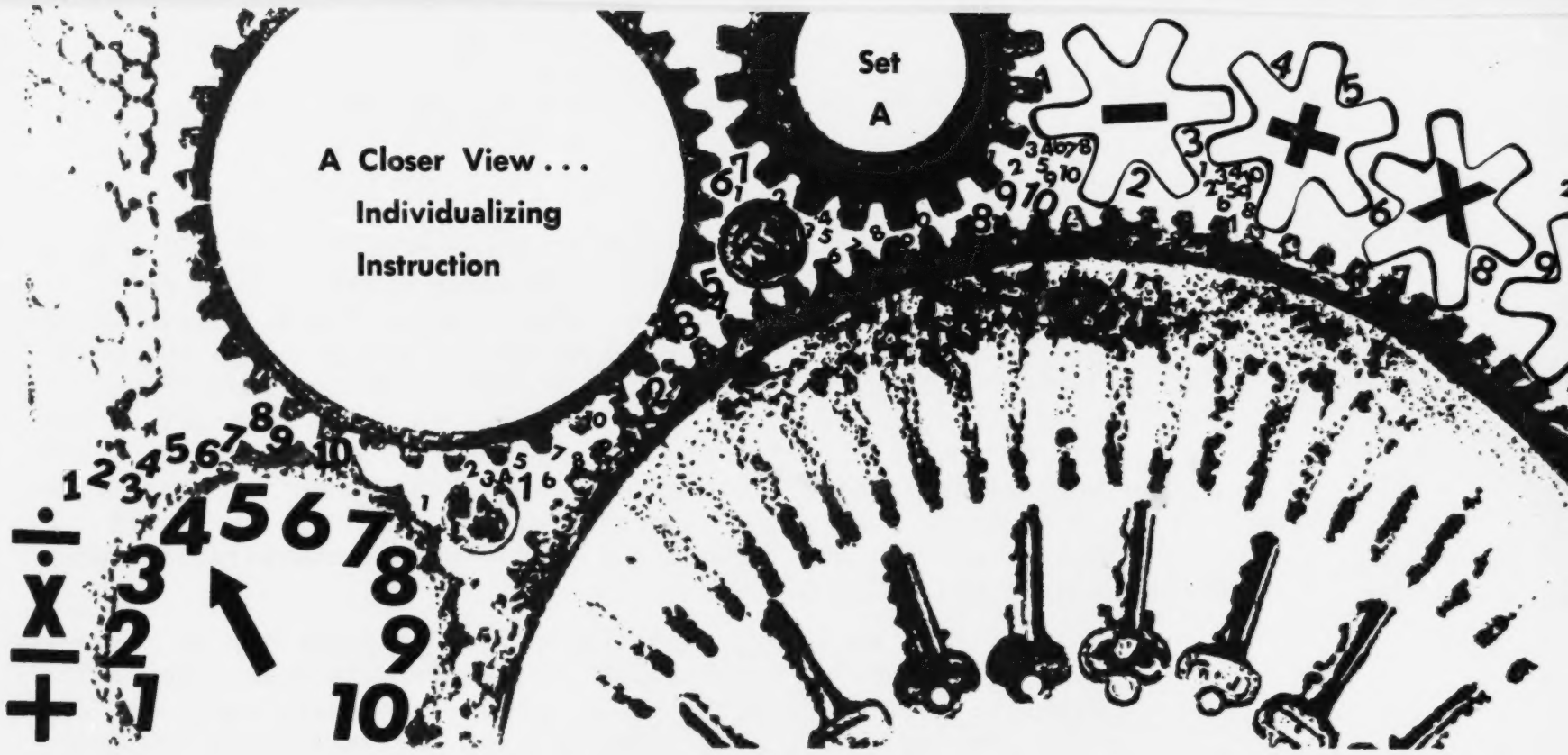

INDIVIDUALIZING INSTRUCTION

By individualizing instruction we mean attempts to organize mathematics programs and instruction in relation to the unique needs and abilities of individual children. This includes, but is not restricted to, plans in which individual pupils work more or less completely independently. It seems apparent that there is no one plan which is best. Provision for individualizing is conditioned in part by school organization, in part by the particular teacher and pupils. The teacher must identify various factors related to pupils' achievement and interest in mathematics, and then decide on appropriate variations in content, materials, method, and time.

What factors are important to consider when individualizing instruction?

Wrigley (1958) was among those who studied the structure of mathematical ability. He concluded that high intelligence is the most important single factor for success in mathematics. He isolated a mathematical group factor which linked the different branches of mathematics, as well as specific verbal, numerical, and spatial factors which affect achievement. When the influence of intelligence was eliminated, verbal ability had little connection with mathematical ability.

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It should be noted that research is variable with respect to its quality; hence, the same degree of confidence cannot be placed in all findings. An attempt has been made to take this fact into consideration in preparing this bulletin.

It has been suggested that the most feasible way of coping with individual differences might be to alter instructional methods to fit the aptitude pattern of the learner. To ascertain whether students high in a given ability achieve better under one method of instruction than under another, King, Roberts, and Kropp (1969) tested 426 fifth and sixth graders after instruction with one of four sets of materials on elementary set concepts. There were significant interactions on inductive-deductive comparisons: it appeared that some students were identified who achieved better when taught inductively, while others achieved more when taught deductively.

Capps (1962) tentatively concluded from a comparison of "superior achievers" and "underachievers" that retardation in mathematics might be related to personal adjustment: perhaps emotional difficulties tend to foster difficulties, and vice versa. Other researchers have also suggested that personality factors may be more important than intelligence in promoting retardation.

Should boys and girls have a different mathematics program?

Jarvis (1964) and Powell, O'Connor, and Parsley (1964) concluded that in general boys scored higher in mathematical reasoning and girls were better in fundamentals, though some conflicting evidence has been presented. Still other studies report no significant achievement differences associated with sex, and most researchers conclude that what little difference exists is not sufficient to influence curriculum decisions.

What is the effect of socioeconomic level on achievement?

There is evidence from research that children from low socioeconomic groups have less mathematical background when they enter school than do children from middle socioeconomic groups. Passy (1964) reported significant differences among third graders, with achievement level increasing as socioeconomic level of the parent increased. Unkel (1966) found that socioeconomic status had a significant effect on achievement in mathematics at all intelligence levels in grades 1 through 9.

How does diagnosis aid in individualizing instruction?

The purpose of diagnosis is to identify strengths as well as weaknesses, and, in the case of weakness, to identify the cause and provide appropriate remediation. As part of the process, there have been many studies which ascertained the errors pupils make. For instance, Roberts (1968) suggested that teachers must carefully analyze the child's method and give specific remedial help.

Most diagnostic tests have been concerned with skill development, but recently the focus has shifted to concept development. Paper-and-pencil tests such as those by Flournoy (1968) and Ashlock and Welch (1966) are not essentially diagnostic, but have implications for those attempting to diagnose pupil understanding.

Bernstein (1959), in a review of the research on remedial teaching of mathematics, noted that every cited experiment used lesson plans based on individual diagnosis as a basic teaching approach. Gray (1966), in reporting on the development of an inventory on multiplication, called attention to the individual-interview

technique pioneered by Brownell: "facing a child with a problem, letting him find a solution, then challenging him to elicit his highest level of understanding."

The Buswell and John (1926) diagnostic study on the four operations, in which the technique of skillful questioning and observing of pupils as they work was employed, has long been of interest. They stress the need to analyze how the child works, which should lead to devising ways of teaching him better methods.

Harvey (1953) reported on diagnostic tests for each operation, and suggested the use of a testing-reteaching-retesting strategy to decrease errors.

How may instruction be effectively individualized?

[For "IPI," see Bulletin A-2.]

Bartel (1966) compared achievement among fourth graders under two treatments: (1) a program of individualized instruction which included content from the "new mathematics," and (2) a "traditional" program, which was not individualized and did not include "new mathematics" content. No significant difference was observed between the two treatments on standardized tests. On a special "Concepts Test," pupils in the individualized program scored significantly higher. Was this difference due to the individualization factor or to the content factor? The design of the investigation does not permit an answer.

Snyder (1967) found no significant differences in achievement between seventh and eighth graders who were allowed to select the mathematical topics they would study and those who could choose from a three-level assignment option. Both groups gained more on reasoning tests and less on skill tests than a third group receiving regular instruction.

McHugh (1959) reported on a two-year differentiated instruction program in grades 4, 5, and 6, in which extensive in-service help was provided to develop a program in which pupils would progress at their own rates, become self-directive and self-correcting, and give mutual help. Significant gains in problem solving were found in grades 5 and 6, and in computational skills in grade 5. The program produced gains "greater than normally expected for the IQ level" in all grades.

Lindgren (1968) reported no significant differences between team learning and learning through conventional teaching in grades 4 and 5, while Wolff (1969) found no significant differences in achievement among third-year pupils in individualized graded or non-graded classrooms.

All in all, there is little substantial evidence to date indicating that programs of individualized mathematics instruction will lead to higher levels of pupil achievement when compared with non-individualized programs.

What types of grouping are effective?

Intraclass grouping to facilitate individualization of reading instruction is a common practice in the elementary school. Evidence on the effectiveness of grouping for mathematics

instruction is conflicting. Part of the conflict is due to grouping on different bases: ability and achievement.

When grouping is based on ability, some studies have shown that homogeneous grouping is especially effective for those with high IQ's (e.g., Provus, 1960; Balow and Ruddell, 1963). Balow and Ruddell, however, found "decreased-range" grouping was more effective than either heterogeneous or homogeneous grouping for most pupils, while Savard (1960) found that such grouping tended to be effective for lower ability pupils and of less advantage for upper ability pupils. Balow and Curtin (1966) reported that grouping by ability did not significantly reduce the range of achievement.

Wallen and Vowles (1960) had each of four sixth-grade teachers use both ability and non-grouping methods for one year. No significant difference was found, though a significant interaction was found between teachers and the methods used. This was not tested in most other studies, and may be the most significant reason for differences in findings.

When grouping is based on achievement, Koontz (1961) found that fourth graders who were heterogeneously grouped achieved significantly higher scores than those homogeneously grouped. Dewar (1963) concluded that providing three intraclass groups benefited high- and low-achieving groups more than did total-class instruction.

Holmes and Harvey (1956) found that there were no significant differences in achievement, attitude, or social structure within the classroom whether pupils were grouped permanently or flexibly (with the topic introduced to all, followed by grouping for further work).

Davis and Tracy (1963) reported that pupils in grades 4, 5, and 6 in self-contained classes scored significantly higher on factors such as verbal and quantitative ability, self-concept, anxiety, and attitude, than did those grouped by both ability and achievement across classrooms at each grade level.

Bernstein (1959) concluded from his review of research that differentiated instruction was more effective than total class instruction, for the general teaching of mathematics as well as for remedial teaching.

What is the effect of acceleration?

In general, acceleration has been reported to be effective for some children. Klausmeier (1963) reported no unfavorable academic, social, emotional or physical correlates of acceleration in fifth graders who had been accelerated from second to fourth grade. Ivey (1965) found that fifth graders who were given an accelerated and enriched program in grade 4 gained significantly more than those receiving regular mathematics instruction.

Jacobs, Berry, and Leinwohl (1965) reported that seventh graders who were in an accelerated program for either three or four

years did significantly better on concepts tests than those who had been accelerated for only one year. There were no significant differences on problem solving tests.

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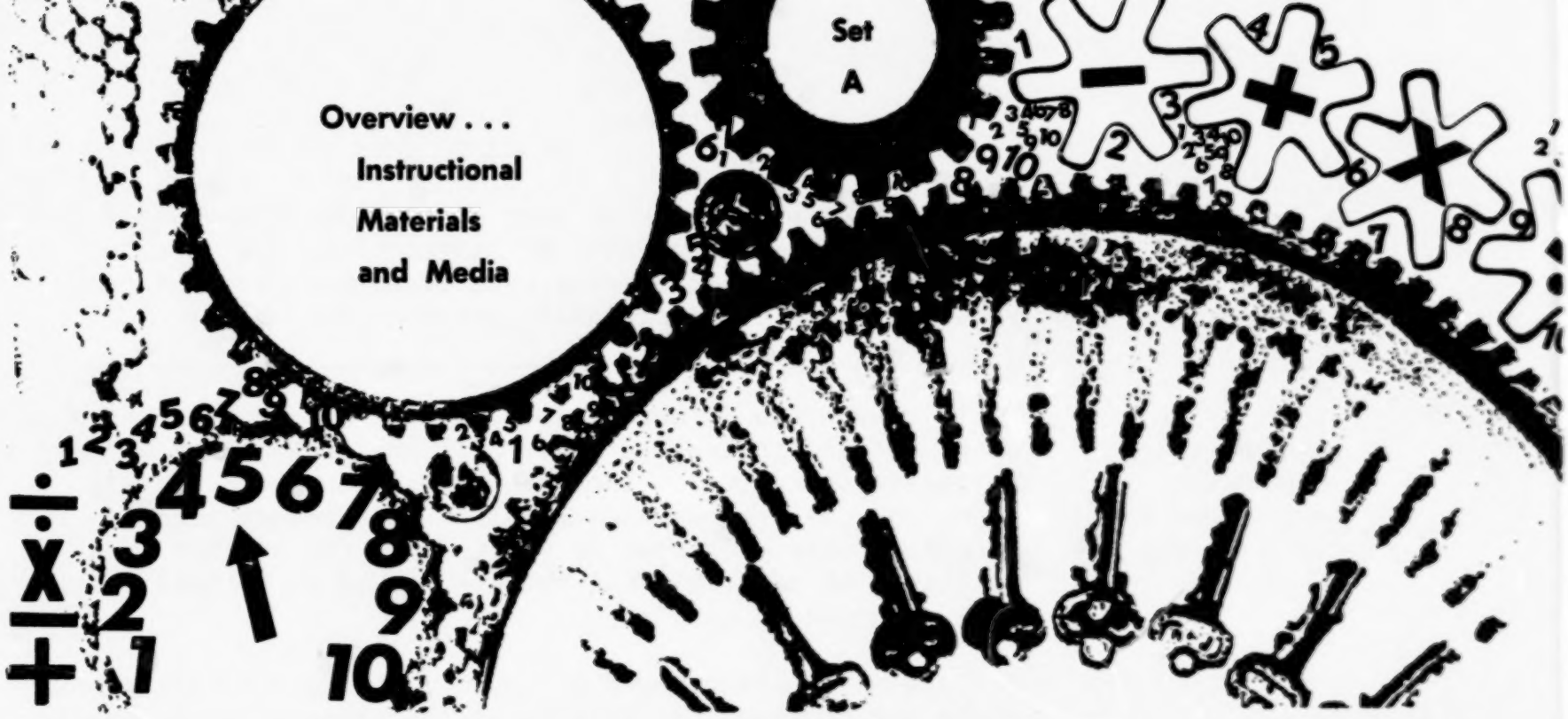

INSTRUCTIONAL MATERIALS AND MEDIA

What may we learn from analyses of mathematics textbooks?

Textbooks have been analyzed from several points of view. Some analyses provide historical perspectives. Others present information on content included in many texts for children and for teachers. Manuals for teachers have also been analyzed in terms of content, objectives, and uses.

Does programmed instruction facilitate achievement?

Research on programmed instruction has shown that it can be used in upper grades to present many topics usually taught at a grade level, as well as topics which are commonly presented at a later grade level. Achievement is usually at least as good as that attained with conventional instruction, but less time is generally required, on the average, when programmed instruction is used. Programmed materials appear to be an effective supplement to the work of the teacher.

What other factors have been investigated with programmed instruction?

Among the other factors which have been investigated with programmed instruction are the effect of various methods of teaching and learning, the effect of materials on pupils with different characteristics, and the amount of time which a teacher spends with individuals when using programmed instruction. When it was used to control the method of presentation, some form of a "guided discovery" approach generally resulted in higher achievement than did teaching by presenting rules. Programmed instruction appeared to be effective for some learners "ordinarily considered less well-adjusted." It has also been found that you may spend a

much greater proportion of your time giving individuals help than you can with conventional instruction; use of programmed materials apparently both frees the time and encourages use of it for individual instruction.

What types of manipulative materials have been found to be effective?

While many types of materials have been studied and found to be effective, it appears that: (1) the learning of mathematics depends more on the teacher than on the materials used, and (2) expensive materials are no better than inexpensive ones developed or provided by the teacher. Use of a variety of materials has not been found to be more effective than use of only one.

Much research has focused on the use of the Cuisenaire materials and program. Especially at the primary level, pupils apparently learn traditional subject matter at least as well as in a conventional program. They also learn some additional concepts and skills. By third grade, however, the effect of earlier teaching of some concepts is less apparent.

How should materials be used?

Strangely, manipulation appears to be less important than we commonly believe. Having pupils manipulate materials themselves has not been found to be more effective than having pupils merely watch the teacher handle them.

Is teaching by television effective?

Television can be used to present key lessons in mathematics, with the teacher using it as an integral part of the program.

How are computers aiding in the instructional process?

Computer-assisted instruction is being explored. Both tutorial programs, in which the computer presents a lesson, and drill-and-practice programs are being used, with promising achievement results.

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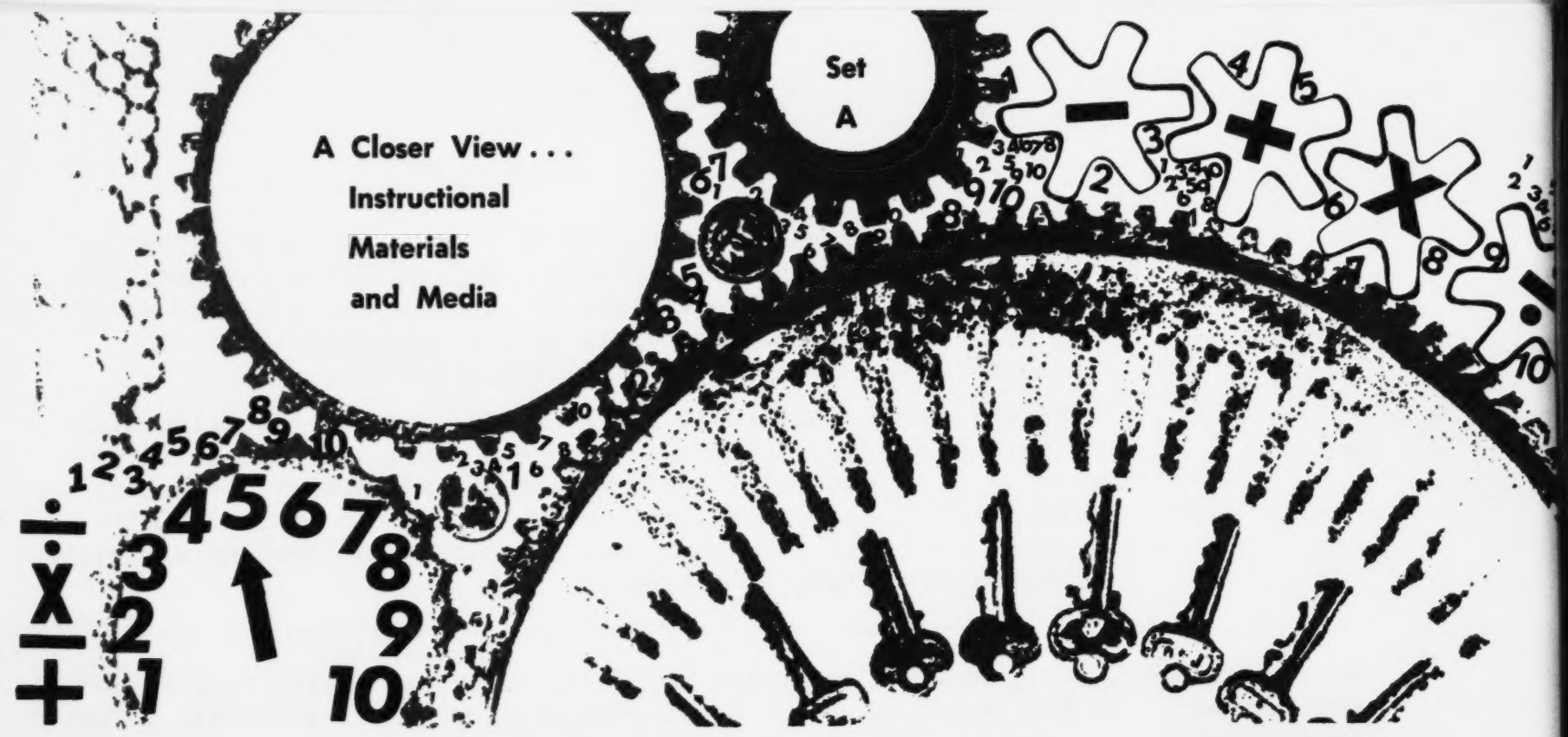

INSTRUCTIONAL MATERIALS AND MEDIA

What may we learn from analyses of mathematics textbooks?

Elementary mathematics textbooks have been analyzed for different purposes and from different bases. One of the most comprehensive analyses is that by Smith and Eaton (1942-43), which includes approximately 200 books used in this country between 1790 and 1940. Their purpose was to study "the basic characteristics and trends of textbooks of the past." Analysis was in terms of the social and economic life of the period, relative emphasis on various aspects of content, the psychological approach, purpose, and scope.

Dooley (1960) studied 153 series of elementary school mathematics textbooks published in the U.S. between 1900 and 1957, attempting to ascertain the effect of research on the content and methods suggested in textbooks. She found that when recommendations were "clear, concise and exact," they were incorporated into many textbooks within five years.

Burns (1960) analyzed ten textbook series and accompanying workbooks and teacher's manuals. He presented specific information

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on the similar content included at each grade level, physical features, and points of emphasis. Folsom (1960) concentrated on manuals, using observations of classroom practice to determine how consistently teachers used suggestions about procedures, enrichment activities, and materials.

Hicks (1968) compared 16 textbooks for teacher education with texts for children, to ascertain the similarities and differences in inclusion of content topics. Marksberry, McCarter, and Noyce (1969) compared cognitive objectives in textbooks with those from research committees and with questions and activities suggested in teacher's manuals.

Does programmed instruction facilitate achievement?

Programmed instruction materials allow each pupil to progress at his own rate. Some studies ascertained the feasibility of using programmed instruction to teach specific content. For instance, Kalin (1962) compared pupils in grades 4, 5, and 6 having IQ's greater than 115 using a programmed text or taught by regular teaching procedures for a two-week unit in equations and inequalities. Differences in achievement were not significant, but 20% less time was spent by those using the program. The idea that the use of programmed materials may result in a decrease in the time which most students must spend on a topic was substantiated in many other studies.

Fincher and Fillmer (1965) reported that fifth graders who used programmed materials on addition and subtraction with fractions achieved significantly greater gains on achievement posttests than pupils using a conventional classroom approach, while retention scores were not significantly different.

In a comparison of a year's program, Banghart and others (1963) found that fourth graders using programmed materials scored significantly higher on comprehension but not on problem solving sections of a standardized test than those receiving regular instruction. They noted that "programmed materials are most effective when used to supplement the classroom teacher."

What other factors have been investigated with programmed instruction?

Neuhouser (1965) found that for eighth graders, programmed materials on exponents were more effective on measures of understanding, ability to transfer, and retention when there was no verbalization of rules, while the program in which pupils were guided to state rules after discovery took longer. The program in which rules were stated for pupils was poorest.

It was reported by Traweek (1964) that fourth graders with poorer personality adjustment scores achieved beyond their expected performance on programmed units on fractions. There were no significant differences in the IQ's of successful and unsuccessful learners.

Teachers using programmed instruction materials devoted 68% of their time to work with individuals, while teachers of conventional classes devoted only 3% of their time to individuals (Goebel, 1966).

What types of manipulative materials have been found to be effective?

Lerch and Mangrum (1965) compiled a list of instructional aids most frequently recommended by the teachers' manuals of nine fourth grade textbooks. Items to be counted, grouped, or described; pocket charts; and number lines were among the aids mentioned most frequently.

Earhart (1964) used an abacus to teach first, second, and third graders whose teachers received in-service help, while other groups received instruction without use of an abacus. On tests of reasoning there were no significant differences, while on tests of fundamentals the group using the abacus performed significantly better. It is difficult to tell whether the abacus or the in-service help was the basis for this difference, however.

Lucas (1967) studied the use of attribute blocks (which are varied in shape, color, and size) in first grade. He found that children trained for 2,000 minutes showed greater ability (1) to conserve cardinally and (2) to conceptualize addition-subtraction relations, than those taught more conventionally in a "modern" program.

Harshman, Wells, and Payne (1962) reported on a study of first graders who were taught for one year by programs with varying content based on either (1) a collection of inexpensive, commercial materials, (2) a commercial set of expensive materials, or (3) materials provided by the teacher. Teachers in the first two instances received in-service training. When significant differences in achievement were observed, they were always in favor of the third program. It was concluded that (1) high expenditure for manipulative materials does not seem justified, and (2) perhaps different materials should be used with different IQ groups.

Much research has been focused on the use of the Cuisenaire materials and program, in attempts to answer the question, "How effective is it?" Crowder (1966) reported that a group of first graders using the Cuisenaire program (1) learned more conventional subject matter and more mathematical concepts and skills than pupils taught by a conventional program; (2) average and above average pupils profited most from the Cuisenaire program; and (3) sex was not a significant factor in relation to achievement, while socioeconomic status was.

Working with first and second graders, Hollis (1965) compared the use of a Cuisenaire program with a conventional approach. He concluded that (1) children learned traditional subject matter with the Cuisenaire program as well as they did with the conventional method, and (2) pupils taught by the Cuisenaire program acquired additional concepts and skills beyond the ones taught in the conventional program.

Brownell (1968) used tests and extensive interviews in an analysis of the effect on underlying thought processes of three mathematics programs, with British children who had studied those programs for three years. He concluded that (1) in Scotland, the Cuisenaire program was in general much more

effective than the conventional program in developing meaningful mathematical abstractions; and (2) in England, the conventional program had the highest over-all ranking for effectiveness in promoting conceptual maturity, with the Dienes and the Cuisenaire programs ranked about equal to each other. Brownell inferred that the quality of teaching was decisive in determining the relative effectiveness of the programs.

Other studies have been concerned with the effect of use of the Cuisenaire program on a particular topic, for shorter periods of time. Lucow (1964) and Haynes (1964) studied use of the program to teach multiplication and division concepts for six weeks in third grade. Lucow attempted to control the effect of prior work in grades 1 and 2. He concluded that the Cuisenaire method was as effective as regular instruction in general, and seemed to operate better in a rural setting, especially with high and middle IQ levels, than in an urban setting. Haynes used pupils who were unfamiliar with the materials; no significant differences in achievement were found between pupils who used the Cuisenaire program and those who did not.

Prior background, length of time, and the specific topic may account for differences in the success of the Cuisenaire program. It has been suggested that it might be more effective in grades 1 and 2, with its effectiveness dissipating during third grade. No body of reported research is available about its effects beyond the third grade level.

Sole (1957) concluded that (1) use of a variety of materials did not "produce better results" than use of only one material, and (2) the learning of mathematics depends more on the teacher than on the materials used.

How should manipulative materials be used?

It may be that having pupils manipulate materials themselves may not be more effective than merely watching the teacher or having no material aid. Jamison (1964) compared instruction in counting in other numeration systems using (1) a large variable-base abacus, (2) a large abacus plus a small abacus for each pupil, and (3) only the chalkboard. There were no significant differences between mean gains.

Toney (1968) also found that a fourth grade group using individually manipulated materials for half a year was not significantly different in achievement from one seeing only a teacher demonstration. And Trueblood (1968) reported no achievement advantage for fourth graders who manipulated materials themselves during a unit on exponents and non-decimal bases.

Is teaching by television effective?

Jacobs and Bollenbacher (1960) reported that after a year of instruction in grade 7 by television or by conventional instruction, significant interaction effects were noted between levels of pupil ability and methods of instruction. Conventional instruction appeared better for those of high ability, the

television method was better at the average ability level, and no significant difference occurred at the below average ability level.

"Patterns in Arithmetic" is a program for grades 1 through 6 which incorporates television instruction. Weaver (1965) summarized a report by Hartung and Suchy on the project at an early stage of its development, noting that there were no significant differences on standardized tests between groups taught by the PIA program or by conventional instruction at the end of the sixth grade after three years of instruction.

Van Engen and Parr (1969) reported that an evaluation of the program in grades 1 and 3 showed that, on both standardized computational and concepts tests, performance of the PIA group "compared favorably" with the norm groups. Attitude of both teachers and pupils toward PIA was also favorable; this has been substantiated by other studies.

How are computers aiding in the instructional process?

Computer-assisted instruction is presently being used in some elementary school mathematics classes. Suppes (1969) has reported extensively on the use of both tutorial and drill-and-practice programs. He found that the drill-and-practice materials result in at least equivalent achievement in less time than it would take the classroom teacher using only conventional methods. The computer also readily collects data on how children are responding, thus facilitating diagnosis of their difficulties as well as increasing our knowledge of how they learn.

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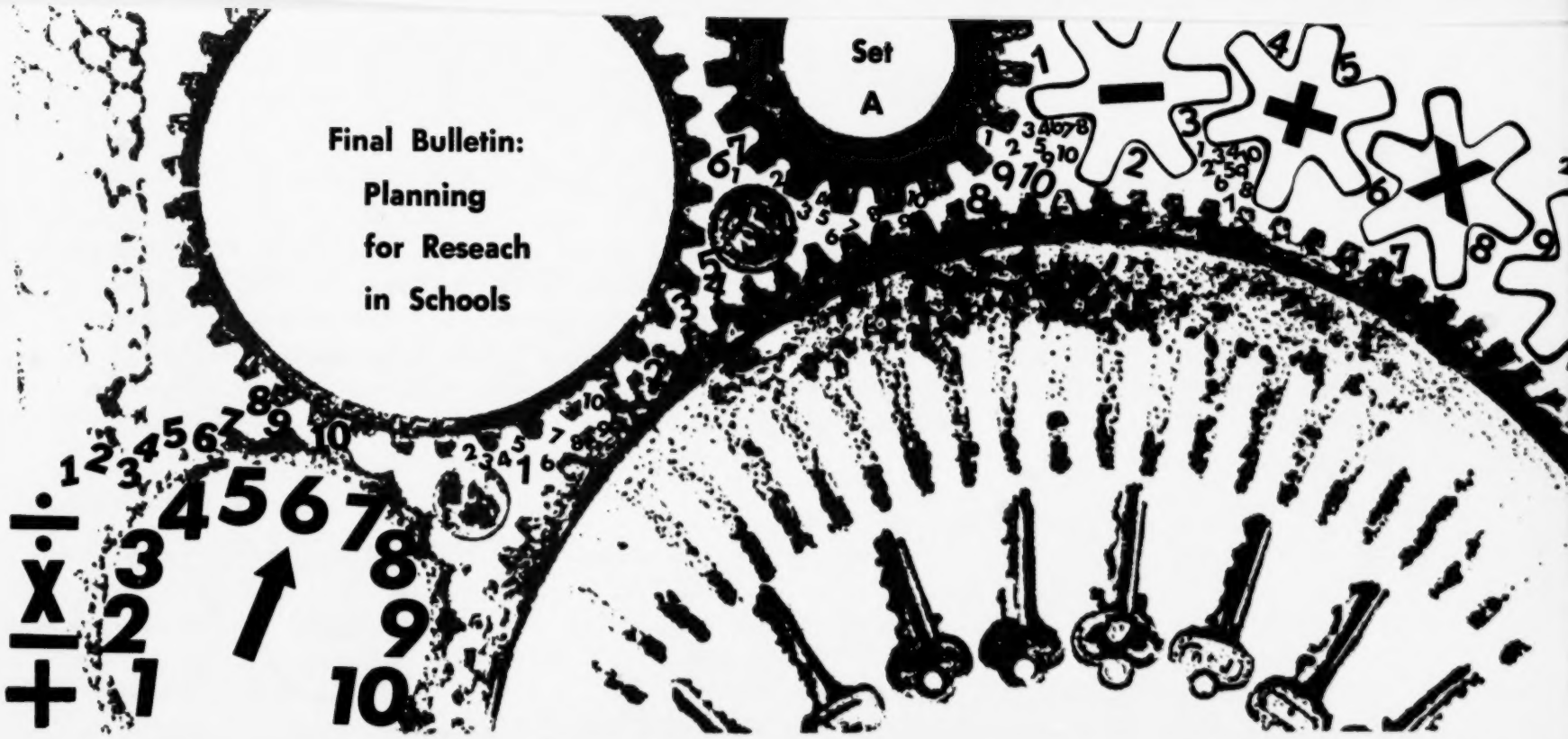


PLANNING FOR RESEARCH IN SCHOOLS

WHAT IS RESEARCH?

Research is controlled inquiry.

In these bulletins, we discuss research on the elementary school mathematics curriculum and research on the teaching and learning of mathematics. The vast majority of this research is product-oriented; there is, however, other research which is theory-oriented. The task of building a theory of the learning of mathematics concepts still lies before us, as Begle (1968) and Glennon (1966) noted.

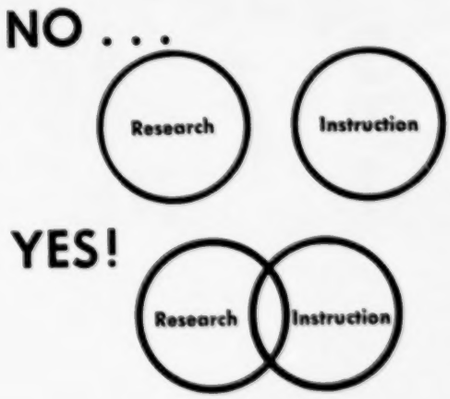
Many of the studies we have cited have involved either experimental or survey research. By experimental we mean research in which the investigator has "manipulated" one or more specified variables, such as two methods of teaching, to measure their effect on another variable, such as achievement or attitude, thus testing a carefully formulated hypothesis or hypotheses. The variables which are manipulated are termed "independent," while those affected and measured are "dependent" variables. Experimental research is very difficult to conduct, because of the need to control the independent variable(s) and many other variables--which must be controlled since we want to interpret the results and generalize beyond the sample in the study. By survey we mean research which attempts to ascertain the characteristics of a population by studying a sample which answers a questionnaire or interview or test.

The material included in this bulletin is a product of the "Interpretive Study of Research and Development in Elementary School Mathematics" (Grant No. OEG-0-9-480586-1352(010), sponsored by the Research Utilization Branch, Bureau of Research, U.S. Office of Education.

The bulletin was prepared by MARILYN N. SUYDAM, The Pennsylvania State University, Project Director, and J. FRED WEAVER, The University of Wisconsin-Madison, Project Consultant. Art by Ed Saffell.

As we continue to discuss "research" in this bulletin, the focus is on experimental research. You should recognize, however, that certain of the things discussed are also applicable to other types of research. We should caution that, despite this focus on experimental studies, we are not thus implicitly stating that such investigations are the only ones which qualify as "true research." Other types of studies also contribute to the improvement of mathematics education.

Research is not independent of instruction. It is derived from and is applied to instruction. Actually, every teacher does a type of "action research" every day--whenever new ideas are tried out. You're constantly trying to find the methods and materials and procedures which will work best for you. You're assessing what pupils have learned, and using what you find out as you plan what to do next. You're concerned with what will help you teach better, or help your pupils learn better. You've been using evaluation, and for some purposes--such as curriculum development --evaluation is vital.



For other purposes, however, research is essential. Research involves more precise controls. In experimental research, we are attempting to secure information which can be generalized to many other teachers and to many different situations. In survey research, we also maintain greater controls than in usual classroom testing--we want a more precise measurement of the status or level of learning.

WHY? Research can provide a foundation on which to make curricular decisions and decisions about how to teach. Nothing has ever been proven by educational research--but it has provided guidelines to aid us in making decisions. It should be noted, however, that not all problems are amenable to research--some decisions must be made on the basis of your philosophy. For instance, research can provide an answer to "Can we teach logic to fourth graders?" but it cannot provide an answer to "Should we teach logic to fourth graders?"

Research has a valid role to play in assessing and improving the quality of instruction. In fact, merely being involved in research helps us to achieve this latter goal. As Pikaart and Berryman (1965) note, "Participating in research and contributing significant ideas was in itself motivating, and it contributed to self-esteem."

Local school systems may at times need to engage in their own research for other reasons. For instance, generalized findings may not be applicable when unique characteristics of the system are considered (e.g., ability level of the pupils).

HOW? First of all, select a question which is important to answer. Then design the study: lay out an overall plan, delimiting the problem to make it researchable. This may be a long-term plan, but don't try to investigate everything at once: order your priorities logically.

You will need to identify and define or describe (1) the independent variable or variables and (2) the dependent variable or variables. You must also identify and control other relevant variables. Suydam (1967) reported that control of variables was one of the two most poorly handled facets of mathematics research studies (sampling was the other one). As Johnson (1966) noted, certain assumptions are made regarding what variables may affect the situation. During an experiment, the groups involved should have common experiences except for the treatment (independent) variables. Then significant differences at the end of the experiment can be attributed to the treatment. Johnson presents an example of an experiment in which many factors are controlled; Wilson (1967) and Worthen (1968) provide other excellent examples of research in which variables are well-controlled.

Some pupil variables may be controlled in one of several ways (Riedesel and Sparks, 1968; Kerlinger, 1964): (1) eliminate the variable as a variable, by studying only a specified subset of the sample; (2) use the statistical procedure of analysis of covariance (but be careful not to "wash out" true differences, as may happen when you apply covariance to a factor of concern); (3) incorporate the factor as another independent variable; (4) match pupil for pupil (this may be difficult, depending on the number of factors on which pupils should be matched); (5) equate on the basis of group means. Or, you can use randomization, where you assume, since pupils are selected by chance, that variables are randomly distributed.

DeVault (1966) and Romberg and DeVault (1967) emphasize our need for realistic research that takes into account the complexity of the classroom setting. On the other hand, if you have a grandiose design that tries to take into account many, many factors, the study will become very complicated. Remember that there is a place to look at small (but not trivial) pieces (Van Engen, 1967).

Select or develop appropriate measuring instruments. Remember especially that "global" or standardized tests are not always appropriate. For example, if you're testing the effect of introducing multiplication in two ways, you'll find that a "global" test has a limited number of items which measure multiplication achievement. The study may result in no significant differences when in fact differences were present--but unmeasured. Instead of a "global" test, a test to measure achievement in multiplication must be constructed.

If two different treatments are to be evaluated, the test must be carefully constructed so it doesn't introduce a bias. Some research has been done in which the test contained a large number of items which only the experimental group would be able to answer (e.g., questions related to a story used to introduce the experimental treatment). Thus the findings of the research favor the experimental group--but not because the pupils did significantly better on the factor being studied.

After you've carefully outlined your research procedures, consider: could I replicate this study, that is, do it over again and expect to get the same results? If you can't answer "yes," replan! Then check your plans with someone who knows research--get professional assistance from your research department or from a university or college, whenever this is possible. This step often makes the difference between good research and a meaningless collection of data, between an answer to your question and no answer. This is the time to clarify questions like "What data should be collected?" and "How will the data be analyzed?" The procedures that are contemplated should not be contemplated independently of consideration of the way in which data are to be collected and analyzed. People have been known to collect data and then wander around trying to find a statistic to use. They don't always find one. In fact, one may not even exist!

Research is improved by being tried out first of all with a pilot study--problems are resolved before they affect your major study. For instance, in doing a survey, the questionnaire should be given to a small group before it is used in the study. A test should be administered to a small group, preferably one much like the group who will be involved in the research. You want to be sure each is valid and reliable, that is, that each measures what it's supposed to measure, consistently.

It is wise to consider the timing of research. Usually it's unwise to plan to begin a study on the first day of school. Beware of other things competing--such as a vacation or other projects which claim priority. Length of time should be appropriate to your problem--remember that most studies can't be done in one day. Also remember that the longer the study, the more problems you may have and the more difficult control becomes.

If you have only two classes at a grade level, the temptation is to have one teacher teach one treatment and the other teach the second treatment. Better yet, have both teachers try both--teaching some pupils by one method and some by the other. This eliminates some confounding, since data for each method can be pooled. The teachers must be doubly careful to not let biases interfere--they must do an honest job with each, despite a special preference for one. The way in which a teacher carries out the research plan is one of the most important factors.

Be sure your sample is appropriate for the population to which you want to generalize your results. There are times when it is reasonable to exclude data for a few children who are very different from the rest of the group, since they may bias the research. Better yet, analyze the data for them separately or differentially.

Whenever appropriate to the design, pupils should be randomly selected and assigned to a treatment. "How many children are needed?" cannot be answered in general: there's a number that will give each study sufficient "power." Remember that it may be wasteful of pupil time to use samples larger than necessary. On the other hand, too small a number raises questions about how representative they are, and how far the findings can be generalized.

There are instances in which it is feasible to conduct research only with intact classes. This situation presents certain problems of research design

which need to be considered. Campbell and Stanley (1963) provide some help on this type of situation.

There is a time to pretest--when you think that pupils have some knowledge of the subject matter. But in other cases, when you can assume that pupils have no knowledge or equivalent knowledge (e.g., when non-decimal bases are introduced in grade 1), a pretest is not necessary. A pilot study using a pretest will indicate whether or not a pretest is necessary in the final study.

It is desirable for teachers involved to keep logs of what was done day by day, as well as anecdotal records of particular incidents and reactions. Then departures from the planned procedures can be noted; these are sometimes useful in interpreting findings. Also, there is a need for somebody to keep a finger on things as the study progresses, to make sure procedures are being followed.

Reporting and disseminating information about research should be carefully done. It is important that others know what you have done and found. Accuracy in reporting is essential, as well as readability. As Weaver (1967) noted, "We can go a long way toward extending the impact of research if each investigator accepts the obligation to report all significant aspects of this work as fully as is necessary to establish the integrity of his research and of the conclusions drawn." The interpretations must be derived from the data--and remember that there is a difference between findings and implications.

We have frequently cited differences which are "significant" or "statistically significant." By this we mean that there is a specified likelihood that such differences would not have occurred by chance. Usually, the level of significance is set at .05, or .01, or .001--thus the results might occur by chance only 5 times in 100, or only 1 time in 100, or only 1 time in 1000. "No significant differences" means that a specified level of significance was not reached--thus the results could occur more frequently by chance. Researchers set a level which seems appropriate to them in terms of the content and design of their study.

In summary, as you plan how to develop and implement your research, you may find these questions helpful:

(1) Is the problem practically and/or theoretically significant?

(2) Is the problem clearly defined?

(3) Is the design appropriate to answer the research question?

(4) Does the design control variables?

(5) Is the sample properly selected for the design and purpose of the research?

(6) Are the measuring instruments valid and reliable?

(7) Are the techniques of analysis of the data valid?

(8) Are the interpretations and generalizations appropriate to the data?

(9) Is the research adequately reported?

WHAT? We look only at mathematics research in these bulletins. Some findings from mathematics research, especially those cited in Set A, might be considered in regard to other phases of the curriculum--and research from other areas may be applicable to the teaching and learning of mathematics. One caution is necessary: don't take findings from one field and assume automatically that they are true for mathematics, or any other. It is important to recognize that conceptual learning is particularly important in mathematics. Mathematics may differ from other areas because there is a body of content (unlike language arts, but like science), which also is sequential; therefore there may be different problems for mathematics than for other areas.

Schools probably do not need to do research on those things on which there is "sound" research evidence already (for instance, on the benefits to be derived from meaningful instruction). There are large variations, however--what may be true in general for large groups may not be true for particular, unique groups. What may be true for one topic may not be true for another.

Teachers should test research findings in their own classrooms (Riedesel, 1968). Remember that just because research says that something was best for a group of teachers in a variety of classrooms, doesn't necessarily mean that it would be best for you as an individual teacher in your particular classroom. For instance, we're beginning to get evidence that there is an interaction between teaching and method. Thus research may show that an inductive approach is "good,"--yet some teachers may not be comfortable with it or can't manage it. An expository approach may be better for those teachers. Teachers have individual differences as well as pupils! In this same way, remember that learning modes of pupils differ, and that not all content lends itself to use of inductive strategies.

Teachers must be careful not to let prior judgments influence their willingness to try out and explore: open-mindedness is important in research. Be willing to investigate. But being open-minded doesn't mean you don't have beliefs about things--just that you don't let beliefs bias the conduct of research.

Often research may be generated by informal exploration that teachers make, which in itself is not research. Do this--but don't call it research; use it to generate hypotheses which can then be tested with research.

AND THEN . . . Research is not an end in itself--it should lead to some kind of action. You decide to change, or not to change; you will accept something, you will reject something. It may lead to other research. Do something as a result of research: incorporate the conclusions of research into your daily teaching.

Non-significant differences can be as important as significant differences--don't be disappointed or think automatically that research has "failed" when no significant differences result. There might in fact be no differences--and the decision is up to you!

In this bulletin, we have been able to give only a glimpse of some of the things which need to be considered as schools conduct research. You may wish to look further into the design and implementation of research as you plan for your own investigations.

Good luck!

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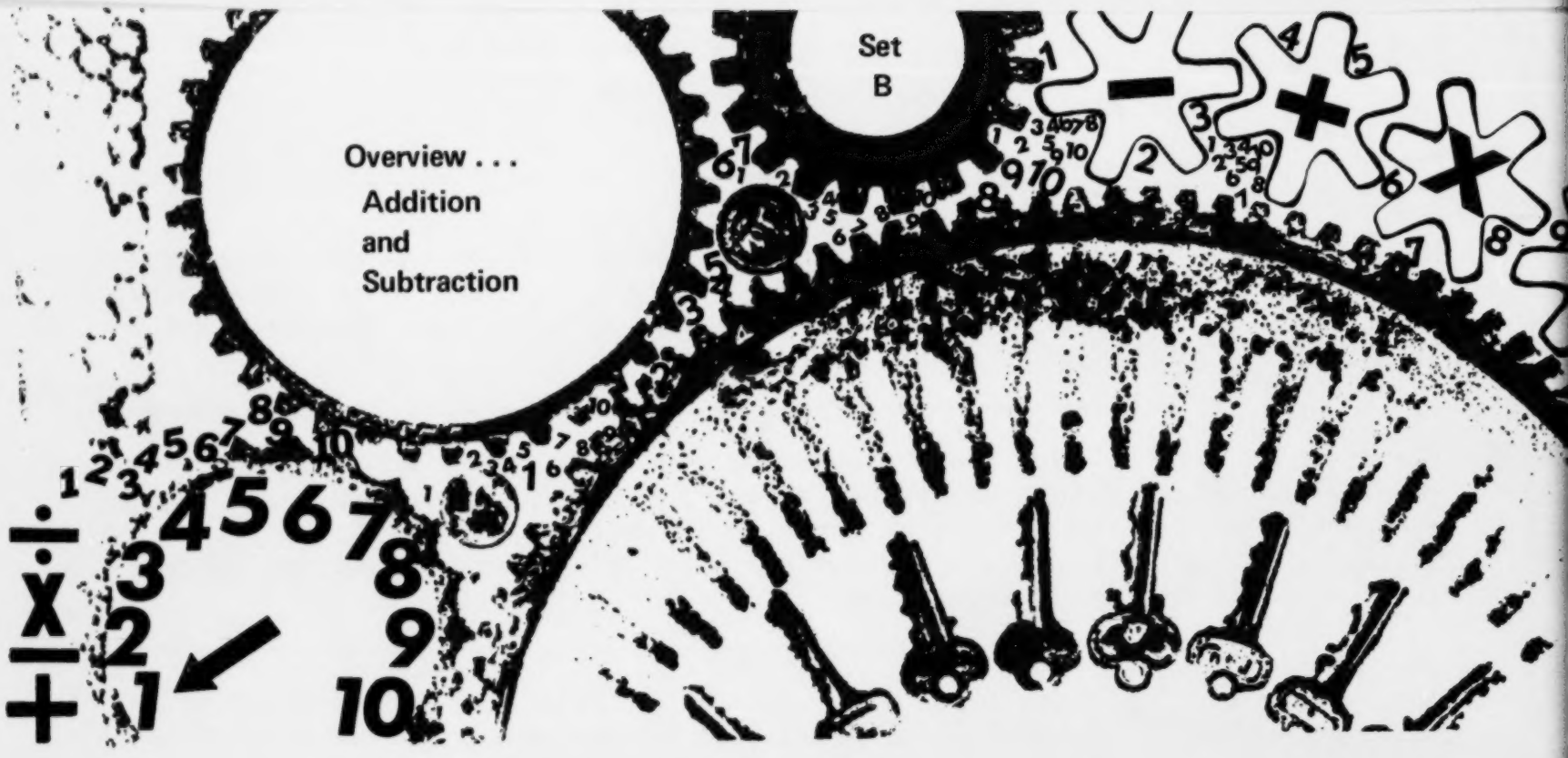


ADDITION AND SUBTRACTION WITH WHOLE NUMBERS

What foundation for addition and subtraction do children have upon entering school?

The ability to count is, of course, of particular importance as a foundation for developing addition and subtraction concepts and skills. Ability to recognize the number of a set without counting and to "conserve numerousness" is also helpful. Surveys have shown that most children can count to at least 19 by the time they enter school, and many can solve addition and subtraction examples which are presented orally.

What is the relative difficulty of addition and subtraction facts?

It has been found in many studies done under a drill method of teaching that:

(1) An addition combination and its "reverse" form tend to be of equal difficulty.

(2) Size of addend is the principal indicator of difficulty.

(3) Combinations with a common addend appear to be of similar but equal difficulty.

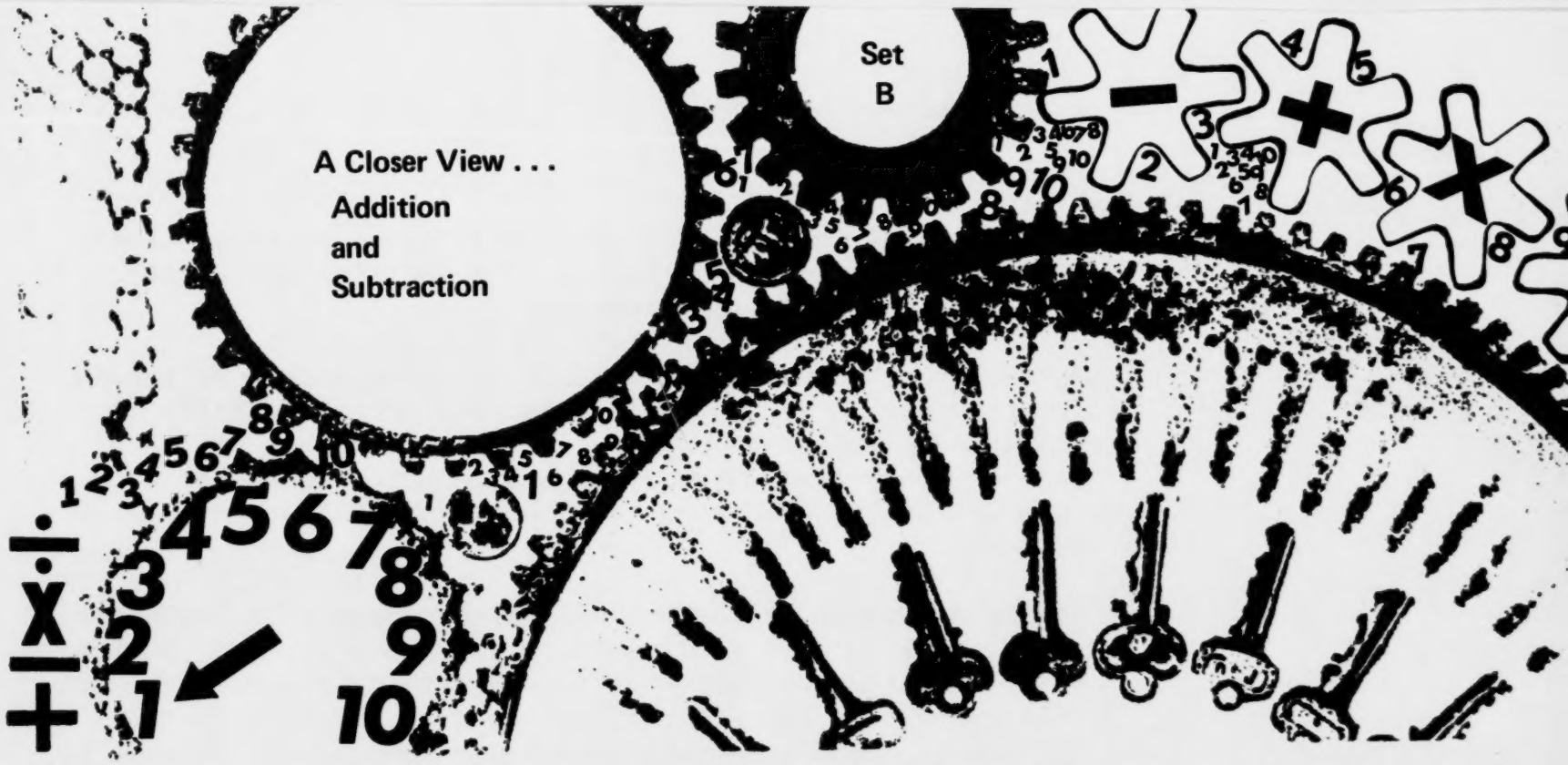
(4) The doubles in addition and those in which 1 is added with a greater number appear to be easiest in addition, while those with differences of 1 or 2 are easiest in subtraction.

<u>However</u>, the order of difficulty seems to be a function of teaching method -- thus research is presently being done to reconsider difficulty level for the meaningful methods in use today.

Should addition and subtraction be introduced at the same time?	In the few studies reported, stress on the relationship between addition and subtraction is found to facilitate understanding, and some increase in achievement has been noted when they are taught together.
What type of problem situation should be used for introductory work with subtraction?	"Take-away" problems are easiest, then "additive" problems, and finally "comparative" problems. Recent research has shown that an approach in which sets are separated into subsets is effective for developing understanding of subtraction situations.
How can number facts be taught effectively?	Experiences with concrete materials have been found to be <u>essential</u> for developing understanding of addition and subtraction concepts. Materials should be appropriate to the child's achievement level and rate of learning. It has been found that children use various ways of obtaining answers to combinations -- guessing, counting, solving using known combinations, and meaningful recall -- and apparently attain mastery only after meaning becomes clear to <u>them</u>.
How should subtraction with renaming be taught?	Decomposition is the renaming procedure used almost exclusively in the United States today. When it is taught meaningfully, understanding and accuracy are better than when it is taught mechanically. Use of the equal additions procedure may lead to even greater accuracy, but possibly at the expense of understanding.
What is the role of drill in teaching addition and subtraction?	<u>Drill must be preceded by meaningful instruction</u>. Accuracy has been and is accepted as a goal in mathematics, but the type of thinking which is developed and the child's facility with the <u>process</u> of thinking is of greater importance than mere recall. Drill and practice should be included at appropriate points; they should be planned to meet the needs of the child.

The material included in this bulletin is a product of the "Interpretive Study of Research and Development in Elementary School Mathematics" (Grant No. OEG-0-9-480586-1352(010), sponsored by the Research Utilization Branch, Bureau of Research, U.S. Office of Education, and conducted at The Pennsylvania State University.

If you would like more information about the research whose findings are cited above, contact MARILYN N. SUYDAM, Project Director, at The Pennsylvania State University, University Park, Pennsylvania, 16802.



Using Research: A Key to Elementary School Mathematics

ADDITION AND SUBTRACTION WITH WHOLE NUMBERS

What foundation for addition and subtraction do children have upon entering school?

As teachers are well aware, a foundation for the development of skills in addition and subtraction is formed long before the first grade. The ability to count is of particular importance: children use counting as a primary means of ascertaining and verifying addition and subtraction facts. The ability to recognize the number of a set without counting is also helpful.

While few experimental studies have been done to determine what <u>can</u> be taught, many surveys have been conducted to ascertain the mathematical ideas and abilities possessed by the pre-school child. The surveys indicate that almost all kindergarten children could count by ones, with most children counting both rotely and rationally to at least 19 (e.g., Bjonerud, 1960; Brace and Nelson, 1965). Less than

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It should be noted that research is variable with respect to its quality; hence, the same degree of confidence cannot be placed in all findings. An attempt has been made to take this fact into consideration in preparing this bulletin.

one-fourth of the children could also count by twos, fives, and tens. Many children could solve addition and subtraction examples in an oral context.

Whether rote counting or rational counting should be taught first is a recurrent question, but has not been explicitly answered by research. Generally, the pre-school child learns to say the number names and then begins to say them in order before he associates the names with sets of objects.

The relationship of the work Piaget has done with "conservation" seems to have applicability to the classroom. Steffe (1968) pointed out that one type of ability possessed by children who do better in first grade mathematics is the ability to "conserve numerousness" -- that is, to be able to specify that "if two sets are matched, one-to-one, the number of objects in each is the same, regardless of the arrangement or rearrangement of the two sets."

At the end of first grade, he administered tests of addition problems and facts to children at four levels of ability to conserve numerousness. Children at the lowest level performed significantly less well on both tests than did children in the upper three levels. At all levels of conservation of numerousness, problems with accompanying physical and pictorial aids seemed to be of about equal difficulty; however, problems with no aids were significantly more difficult. Problems in which one of two sets is described as being moved to the other were also significantly easier than problems in which the two sets are static.

Steffe concluded that ability to conserve numerousness thus seems to be related to achievement on addition problems.

LeBlanc (1968) reported on a parallel study with subtraction problems and facts. Children who were in the highest level of conservation of numerousness performed better than children in the lowest two levels. Problems accompanied by aids and those with a description of movement were significantly easier than other types of problems. LeBlanc suggested that a test of conservation of numerousness would provide a basis for a readiness test for first graders.

What is the relative difficulty of addition and subtraction facts?

At one time, especially when stimulus-response theories of learning were prevalent, there was great interest in ascertaining the relative difficulty of the basic number facts or combinations -- e.g., $5 + 2 = 7$, $9 + 6 = 15$, $8 - 3 = 5$, $17 - 9 = 8$. Textbook writers as well as classroom teachers used the results of such research to determine the order in

which facts would be presented. The assumption was that if the combinations were sequenced appropriately, the time needed to memorize them could be reduced.

The relative difficulty of the combinations generally was derived from a study of either (1) the number of errors made on each combination, (2) reaction time, (3) retention after a period of non-use, (4) the number of repetitions needed for immediate recall during initial learning, or (5) familiarity with combinations among children entering school. The varying procedures are, in part, the reason for lack of agreement among the studies.

Nevertheless, some common findings were evident which, despite the age of the studies, may in part still be applicable (e.g., MacLatchy, 1933; Washburne and Vogel, 1928; Wheeler, 1939):

(1) An addition combination and its "reverse" form tend to be of equal difficulty.

(2) Size of addend is the principal indicator of difficulty, rather than size of sum.

(3) Combinations with a common addend appeared to be of similar but not equal difficulty.

(4) The "doubles" in addition and those in which 1 is added with a greater number appear to be easiest in addition, while those with differences of 1 or 2 are easiest in subtraction.

Swenson (1944) questioned whether results on relative difficulty obtained under repetitive drill-oriented methods of learning are valid when applied in learning situations not so definitely drill-centered. When second graders were taught by drill, by generalization, and by a combined method, it was found that the order of difficulty seemed to be, at least in part, a function of teaching method. Thus research which aims at establishing the difficulty of arithmetic skills and processes should probably do so in terms of a clearly defined teaching and learning method.

Recently, Suppes (1967) has been interested in using the data-gathering potential of the computer to explore the relative difficulty of mathematical examples, including the basic facts. A drill-and-practice program which presents addition and subtraction combinations has been used as the vehicle to determine a suggested order of presentation and amount of practice.

Should addition and subtraction be introduced at the same time?

It is somewhat surprising, considering how frequently this question is asked, to find that there has been little research on the topic. Early studies (such as Brownell, 1928) found that higher achievement resulted when addition and

subtraction facts were taught together. Spencer (1968) recently reported that there may be some intertask interference, but emphasis on the relationship facilitates understanding.

Research has generally found that the subtraction combinations are harder for children to learn than those in addition, even when addition and subtraction are taught together.

What type of problem situation should be used for introductory work with subtraction?

Gibb (1956) explored ways in which pupils think as they attempt to solve subtraction problems. In interviews with 36 second graders, she found that pupils did best on "take-away" problems and poorest on "comparative" problems. For instance, when the question was, "How many are left?", the problem was easier than when it was, "How many more does Tom have than Jeff?". "Additive" problems, in which the question might be, "How many more does he need?", were of medium difficulty and took more time. She reported that the children solved the problems in terms of the situation, rather than conceiving that one basic idea appeared in all applications.

Schell and Burns (1962) found no difference in performance on the three types of problems. However, "take-away" situations were considered by pupils to be easiest -- thus they are generally considered first in introductory work with subtraction.

Coxford (1966) and Osborne (1967) found that an approach using set-partitioning, with emphasis on the relationship between addition and subtraction, resulted in greater understanding than the "take-away" approach. Consideration of this finding is important to those who want to develop set-subset concepts as a strand in the curriculum.

How can number facts be taught effectively?

Brownell (1928, 1941) and McConnell (1934) found that pupils use various ways of obtaining answers to combinations -- guessing, counting, and solving from known combinations, as well as immediate recall. Brownell stated, "Children appear to attain 'mastery' only after a period during which they deal with procedures less advanced (but to them more meaningful) than automatic responses."

In general, experiences with concrete materials provide an essential base for developing understanding of addition and subtraction concepts. Encouraging pupils to use drawings as well as objects may help those having difficulty learning combinations (Brownell, 1928).

Generally, researchers have concluded that understanding is best facilitated by the use of concrete materials; followed by semi-concrete materials such as pictures, and finally by the abstract presentation with words and/or numerals.

Gibb (1956) also found that abstract contexts were poorest. She reported, however, that pupil performance was better on subtraction examples presented in a semi-concrete context, rather than with concrete materials. Nevertheless, she noted, "Children have less difficulty solving problems if they can manipulate objects or at least think in [the] presence of objects with which the problems are directly associated than when solving problems wholly on a verbal basis."

Klausmeier and Feldhusen (1959) are among those who have found that curriculum materials should be appropriate to the learner's achievement level and rate of learning. Then both initial achievement and retention are not significantly different across intelligence levels.

Transfer was studied by Olander (1931). Pupils who had studied only 55 addition and 55 subtraction combinations (omitting the "reverse" forms) were also able to answer most of the 90 which they had not studied, doing almost as well as those who studied all 200 combinations.

How should subtraction with renaming be taught?

Over the years, researchers have been very concerned with procedures for teaching subtraction involving renaming (once commonly called "borrowing"). The question of most concern has been whether to teach subtraction by equal additions or by decomposition.

How do <u>you</u> do this example?

$$\begin{array}{r} 91 \\ -\ 24 \\ \hline 67 \end{array}$$

You're using <u>decomposition</u> if you do it this way:

11 - 4 = 7 (ones); 8 - 2 = 6 (tens)

If you do it this way, you're using <u>equal additions</u>:

11 - 4 = 7 (ones); 9 - 3 = 6 (tens)

In a classic study, Brownell (1947; Brownell and Moser, 1949) investigated the comparative merits of two algorithms (decomposition and equal additions), in combination with two methods of instruction (rational or meaningful, and mechanically):

	rational (meaningful)	mechanical
decomposition	a	b
equal additions	c	d

He found that, at the time of initial instruction:

(1) Rational decomposition [a] was better than mechanical decomposition [b] on measures of understanding and accuracy.

(2) Rational equal additions [c] was significantly better than mechanical equal additions [d] on measures of understanding.

(3) Mechanical decomposition [b] was not as effective as either equal additions procedure [c or d].

(4) Rational decomposition [a] was superior to each equal additions procedure [c, d] on measures of understanding and accuracy.

It was concluded that whether to teach the equal additions or the decomposition algorithm depends on the desired outcome.

In recent years, the decomposition procedure has been used almost exclusively in the United States, since it was considered easier to explain in a meaningful way. However, some question has recently been raised about this: with increased emphasis in many programs on properties and on compensation in particular, the equal additions method can also be presented with meaning. For instance, pupils are learning that:

(a) $9 - 3 = \square$ means that $\square + 3 = 9$

or $3 + \square = 9$

They are learning that:

(b) $7 - 4 = 3$ is equivalent to

$$(7 - 4) + 2 = 3 + 2$$

Development of such ideas should facilitate the teaching of the equal additions procedure. Whether there will again be

a shift toward wider use of this procedure remains to be seen. Evidence from other studies indicates that use of it leads to greater accuracy.

What facilitates the learning of more complex skills?

Brownell (1947) studied the use of a crutch such as

$$\begin{array}{r} {}^{4\prime}\not{5}6 \\ -\ 39 \\ \hline 17 \end{array}$$

This seemed to facilitate understanding, but attempts to have pupils stop using the crutch were not wholly successful. Some persons suggest that this crutch should only be taught when it is needed.

Overman (1930) found that if pupils were taught to generalize about the renaming procedures in two-place addition and subtraction, they were able to do three-place examples. This was less time-consuming than having the teacher present two-place and then three-place examples separately.

Ekman (1967) reported that when third graders manipulated materials before presentation of an addition algorithm, both understanding and ability to transfer increased. Use of materials was better than use of only pictures before introduction to the algorithm, or development of the algorithm without either aid.

What is the role of drill in teaching addition and subtraction?

Discussions on the teaching of mathematics in the primary grades once centered on whether programs should consist of isolated, repetitive drill or of an integrated approach involving the presentation of interrelated ideas. Prior to the 1930's, much research was done on the effectiveness of various types of drill. For instance, Knight (1927) reported on a successful program of drill in which the distribution of practice on basic facts was carefully planned -- no facts were neglected, but more difficult combinations were emphasized.

Accuracy has been and is accepted as a goal in mathematics, and it is in an attempt to meet this goal that drill is stressed. In a series of articles, Wilson advocated no less than 100% mastery. He showed that, with a carefully planned set of materials, the goal was not as unattainable as some persons believed it to be.

Many other studies have shown that drill per se is not effective in developing mathematical concepts. Programs stressing relationships and generalizations among the

addition and subtraction combinations were found to be preferable for developing understanding and the ability to transfer (McConnell, 1934; Thiele, 1938). This has been supported by many studies since that time.

Brownell and Chazal (1935) summarized their research work with third graders by stating that drill must be preceded by meaningful instruction. The type of thinking which is developed and the child's facility with the process of thinking is of greater importance than mere recall. Drill in itself makes little contribution to growth in quantitative thinking, since it fails to supply more mature ways of dealing with numbers.

Pincus (1956) also found that whether drill did or did not incorporate an emphasis upon relationships was not significant, when drill followed meaningful instruction.

Should non-paper-and-pencil practice be provided?

Many mathematical problems which arise in everyday life must be solved without pencil and paper. Providing a planned program of non-paper-and-pencil practice on both examples and problems has been found to be effective in increasing achievement in addition and subtraction, as for other topics in the curriculum (Flournoy, 1954). Other researchers have suggested that certain "thought processes" which are especially suited to such practice should be taught. For instance, a left-to-right approach to finding the sum or difference is useful, rather than the right-to-left approach used in the written algorithm. "Rounding," using the principle of compensation, and renaming are also helpful. Increased understanding of the process may result.

Should children "check" their answers?

The answers which research has provided to this question are not in total agreement. We encourage children to check their work, since we believe that checking contributes to greater accuracy. There is some research evidence to support this belief.

However, Grossnickle (1938) reported data which should be considered as we teach. He analyzed the work of 174 third graders who used addition to check subtraction answers. He found that pupils frequently "forced the check," that is, made the sums agree without actually adding; in many cases, checking was perfunctory. Generally, there was only a chance difference between the mean accuracy of the group of pupils when they checked and their mean accuracy when they did not check.

What does this indicate to teachers? Obviously, children must understand the purpose of checking -- and what they must do if the solution in the check does not agree with the original solution.

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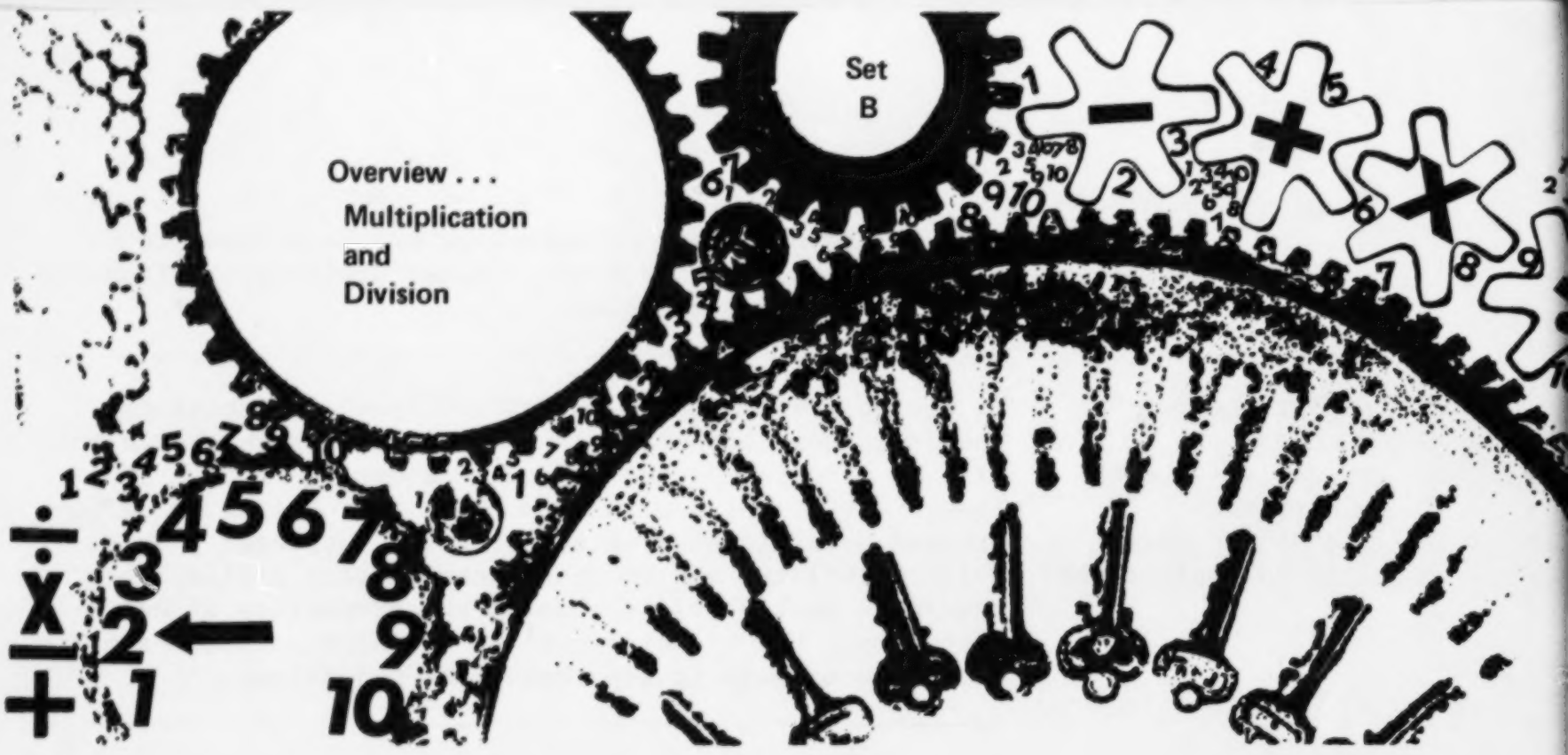


MULTIPLICATION AND DIVISION WITH WHOLE NUMBERS

Should children be encouraged to memorize basic multiplication facts?

Of course children should achieve immediate recall of the basic facts -- at an appropriate time in the learning sequence. Understanding of the nature of multiplication should precede work which focuses on such memorization, however. Use of properties of multiplication will help pupils in this learning.

How should multiplication be conceptualized for children?

Multiplication usually has been conceptualized in terms of the addition of equal addends. Arrays are also suggested as a way of representing multiplication, though little research has been done using them. Cartesian-product problems appear to be more difficult for young children to conceptualize.

Is attention to distributivity helpful in early work with multiplication?

Emphasis on distributivity is especially effective in promoting transfer and retention. Research on this adds further support to a growing body of evidence on advantages to be expected from instruction which emphasizes understanding. The "pay-off" may not always be evident in immediate achievement of skills, but rather in relation to factors such as comprehension, transfer, and retention.

What has been found about other approaches to early work with multiplication?

Do you usually introduce multiplication with verbal problems? If you do this, and then guide pupils in developing the multiplication fact from each problem (by counting, using pictures and diagrams, adding, and using the number line), recall and retention of the facts should be facilitated. Such an inductive approach, where each pupil can

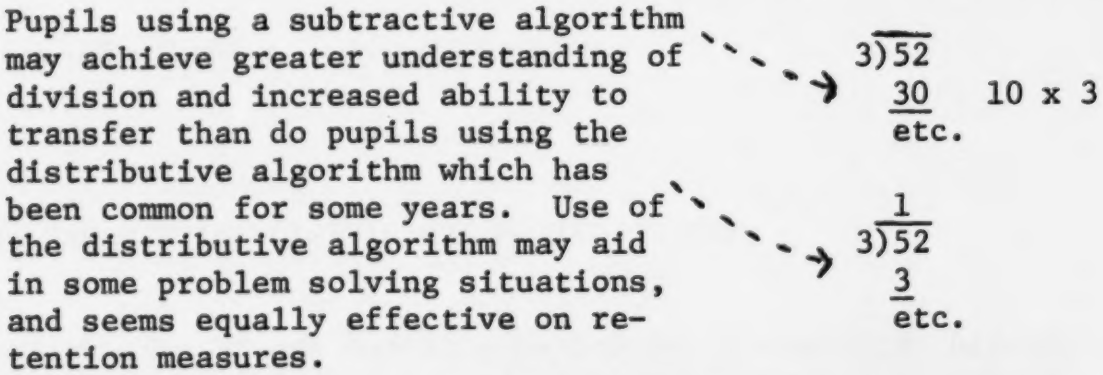
work at his own level of maturity, has been shown to be better than one in which the teacher presents the facts to the pupil through examples.

What things contribute to pupils' success with more advanced work in multiplication?

If you only want pupils to achieve speed and accuracy, then readiness for work with two-place factors should consist of practice on the 100 multiplication facts. If, however, you want pupils to achieve the objectives of increased understanding of the process, increased problem solving ability, and increased computation skills, then readiness work should emphasize the properties of multiplication. Use of the algorithm in which partial products are shown appears to aid these same objectives.

Which division algorithm should be used?

Pupils using a subtractive algorithm may achieve greater understanding of division and increased ability to transfer than do pupils using the distributive algorithm which has been common for some years. Use of the distributive algorithm may aid in some problem solving situations, and seems equally effective on retention measures.



What is the most effective method of teaching pupils to estimate quotient digits?

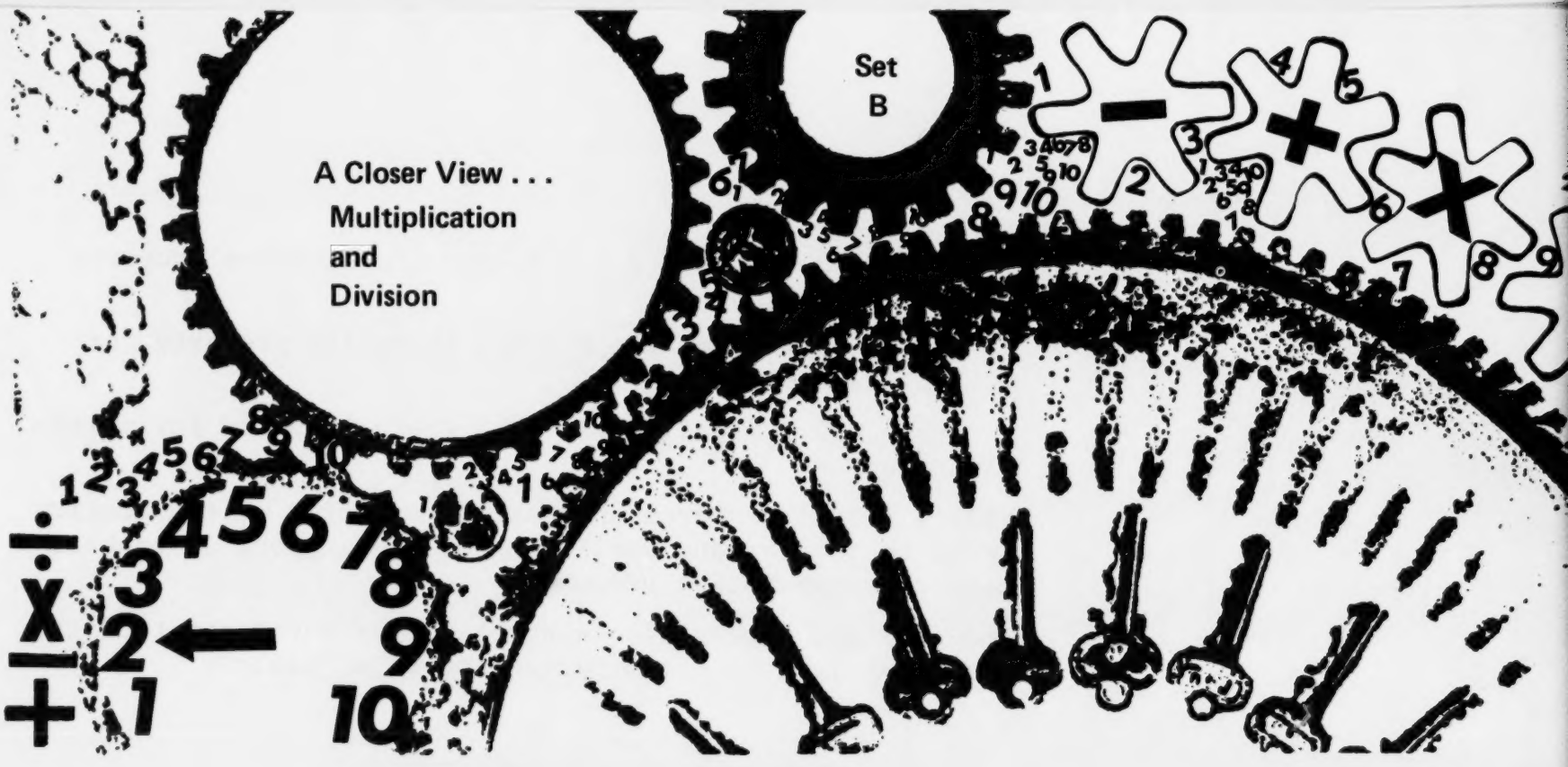
If success on first trial is the criterion, then "round-both-ways" (42 → 40, 47 → 50) would be recommended. However, corrections must be made by either increasing or decreasing the estimate. With the "round-down" method (42 → 40, 47 → 40) the estimate is corrected by decreasing it, while with the "round-up" method (42 → 50, 47 → 50) the estimate is corrected by increasing it. This last method parallels the procedure used in the subtractive algorithm.

What is the role of measurement and partition situations in teaching division?

Partition problems appear to be more difficult than measurement problems. Use of the subtractive algorithm for measurement situations and the distributive algorithm for partition situations has been suggested.

The material included in this bulletin is a product of the "Interpretive Study of Research and Development in Elementary School Mathematics" (Grant No. OEG-0-9-480586-1352(010), sponsored by the Research Utilization Branch, Bureau of Research, U.S. Office of Education, and conducted at The Pennsylvania State University.

If you would like more information about the research whose findings are cited above, contact MARILYN N. SUYDAM, Project Director, at The Pennsylvania State University, University Park, Pennsylvania, 16802.



MULTIPLICATION AND DIVISION WITH WHOLE NUMBERS

Should children be encouraged to memorize basic multiplication facts?

At an appropriate time in the learning sequence it is desirable that children strive to achieve immediate recall of basic multiplication facts (3 x 5 = 15, 6 x 4 = 24, 7 x 8 = 56, 9 x 9 = 81, etc.).

Findings from a comprehensive investigation with children in grades three to five by Brownell and Carper (1943) suggest that activities and experiences which contribute to pupils' understanding of the mathematical nature of multiplication should precede work which focuses on memorization of facts.

Teachers know that the number of specific basic facts to be memorized is reduced substantially if pupils are able to apply the properties of multiplication illustrated by the following examples:

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The bulletin was prepared by MARILYN N. SUYDAM, The Pennsylvania State University, Project Director, and J. FRED WEAVER, The University of Wisconsin-Madison, Project Consultant. Art by Ed Saffell.

It should be noted that research is variable with respect to its quality; hence, the same degree of confidence cannot be placed in all findings. An attempt has been made to take this fact into consideration in preparing this bulletin.

(a) 3 x 5 = 15 and 5 x 3 = 15. (Commutative property of multiplication)

(b) 8 x 1 = 8 and 1 x 8 = 8. (Identity property for multiplication)

(c) 7 x 0 = 0 and 0 x 7 = 0. (Zero property for multiplication)

Hall's (1967) research on teaching selected multiplication facts to third-grade pupils appears to support an emphasis upon the commutative property.

Brownell and Carper also suggested that development of the facts may lead to the organization of a "table":

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	8	10	
3	0	3	6	9	12	15	

This can aid in the identification of patterns and relationships; pupils can find answers to such questions as:

-- If 1 is a factor, what pattern is true?

-- If 5 is a factor, what digit will be in the units place in the product?

-- If one factor is even, will the product be odd or even?

Ascertaining the relative difficulty of the multiplication facts was once a matter of great concern, based on the assumption that there is a fixed rank for each. Little commonality of levels of difficulty was evident among the studies, however, since this is apparently a function of (1) whether pupils are studied at the time of initial learning, or later; (2) the order and organization of the facts; and (3) the method of teaching, whether meaningful, with emphasis on relationships, or drill-oriented. Thus we need to ask, "Difficulty level for whom? at what age? under what method of instruction?"

Two findings that were frequently cited in the early studies (conducted under a drill approach) were that combinations involving zero presented difficulty, and that the size of the product was positively correlated to difficulty. Whether these remain true today, where a more meaningful teaching approach is used, has not been ascertained by research, but nevertheless should be considered by the teacher.

How should multiplication be conceptualized for children?

Traditionally multiplication of whole numbers has been conceptualized for children in terms of the addition of equal addends. For instance, "4 x 7" has been interpreted to mean "7 + 7 + 7 + 7." But there are logical difficulties inherent in this interpretation when the first factor in a multiplication example is 0 or 1.

Some recent research has investigated the feasibility of using other conceptualizations of multiplication. One of these interpretations, which is independent of addition, is based upon the following relationship: if set A has a members and set B has b members, the Cartesian product of sets A and B has a x b members. Hervey (1966) reported that second-grade pupils had significantly greater success in solving, conceptualizing, and visually representing equal-addends problems than Cartesian-product problems. Cartesian-product problems were conceptualized and solved more often by high achievers than by low achievers, more often by boys than by girls, and more often by pupils with above-average intelligence. Hervey was not able to determine the extent to which her findings may be influenced by the nature of prior instruction or by differences inherent in the mathematical nature of the two conceptualizations.

Another conceptualization of multiplication may be associated with rectangular arrays -- either independent of or in conjunction with Cartesian products. At the third-grade level Schell (1964) investigated achievement of pupils who used array representations exclusively for their introductory work with multiplication, as compared with pupils who used a variety of representations. He found no conclusive evidence of a difference in achievement levels.

Is attention to distributivity helpful in early work with multiplication?

We know, for example, that 3 x (4 + 7) = (3 x 4) + (3 x 7). This is an instance of the distributive property of multiplication over addition which (in one form or another) is used to some extent in contemporary programs of mathematics instruction. Specific instances of this property often are illustrated with arrays.

Although Schell (1964) reported some findings regarding third-graders' ability to use distributivity, his observations were based upon a very limited amount of instruction: two introductory lessons. Such findings are tenuous at best.

From a more comprehensive investigation with third-grade pupils and their beginning work with multiplication, Gray (1965) found that an emphasis upon distributivity led to "superior" results when compared with an approach that did not include work with this property. The superiority was statistically significant on three of four measures: posttest of transfer ability, retention test of

multiplication achievement, and retention test of transfer. On the remaining measure -- posttest of multiplication achievement -- children who had worked with distributivity scored higher than those who had not, but the difference was not statistically significant.

Gray's findings add further support to a growing body of evidence on advantages to be expected from instruction which emphasizes mathematical meaning and understanding. The "pay-off" may not always be particularly evident in terms of skills-achievement immediately following instruction. Rather, the pay-off is much more clearly evident in relation to factors such as comprehension, transfer, and retention.

What has been found about other approaches to early work with multiplication?

Fullerton (1955) compared two methods of teaching the "easy" multiplication facts to third-graders: (1) an inductive method by which pupils developed multiplication facts from word problems, using a variety of procedures; and (2) a "conventional" method which presented multiplication facts to pupils without involving them in the development of such facts. In this instance a significant difference in favor of the inductive method was found on a measure of immediate recall of taught facts as well as on measures of transfer and retention.

In another investigation Haynes (1964) concluded that the Cuisenaire approach to multiplication (based upon Gattegno's texts) was no more effective with third-graders than was a "conventional" method exemplified by a well-known and often-used arithmetic textbook published in 1959. [Research on the Cuisenaire approach within other contexts is reported in Bulletin A-5.]

What things contribute to pupils' success with more advanced work in multiplication?

On the basis of multiple criteria, Schrankler (1967) evaluated the relative effectiveness of two algorithms for teaching multiplication with whole numbers to fourth grade pupils. As interacting factors, he considered (1) three intelligence levels and (2) two readiness backgrounds. From a variety of findings Schrankler concluded that methods using general ideas based on the structure of the number system are more successful than other methods investigated in achieving the objectives of increased computational skills, understanding of processes, and problem solving abilities associated with the multiplication of whole numbers between 9 and 100.

What is the difficulty level of division combinations?

Little research has been done on the difficulty level of the basic division facts, but great attention has been given to the difficulties inherent in the algorithm. Osburn (1946) noted 41 levels of difficulty for division

examples with two-digit divisors and one-digit quotients. Pupils' ability to divide with two-figure divisors has been found to involve a considerable variety of skills varying widely in difficulty (Brownell, 1953; Brueckner and Melbye, 1940). Examples in which the apparent quotient is the true quotient (as in $43\overline{)92}$ are (of course) much easier than those requiring correcting (such as $43\overline{)81}$), with difficulty increasing as the number of digits in the quotient increases.

Which is it better to teach: the subtractive or the distributive form of the division algorithm?

During the 1940's and 1950's, the division algorithm typically taught in elementary school mathematics was:

```
      2
23)552          First think
  46            '2's in 5?'
  92
  etc.
```

(Some people refer to this as the distributive algorithm.)

Today, a multiplicative and subtractive approach to the division algorithm has come back into use:

```
23)552
  230    10 x 23
  322
  230    10 x 23
   92
  etc.
```

In one investigation comparing use of the conventional (or distributive) and the subtractive forms, Van Engen and Gibb (1956) reported that there were some advantages for each. They evaluated pupil achievement in terms of understanding the process of division, transfer of learning, retention, and problem solving achievement. Among their conclusions were:

(1) Children taught the subtractive method had a better understanding of the process or idea of division in comparison with the conventional method used. Use of this algorithm was especially effective for children with low ability. Those with high ability used the two methods with equivalent effectiveness.

(2) Children taught the conventional (distributive) method achieved higher problem solving scores (for the type of problem in the study).

(3) Use of the subtractive method was more effective in enabling children to transfer to unfamiliar but similar situations.

(4) The two procedures appeared to be equally effective on measures of retention of skill and understanding. This seems to be more related to teaching procedures, regardless of the method of division.

In another study of the division algorithm with twelve fourth grade classes, Dawson and Ruddell (1955) compared the effectiveness of (1) "common textbook practices" and (2) a procedure in which division was presented as "a special case of subtraction." The second procedure also stressed "meaningful" instruction through much use of discussion and manipulative materials. The investigators concluded that this latter approach resulted in significantly higher achievement (immediately following instruction as well as after a retention period of seven weeks), and increased ability to solve examples in a new situation. It also helped pupils to develop greater understanding of division and its interrelationships with subtraction, multiplication and addition than did the "common textbook practices" approach. Whether these findings were related primarily to the emphasis on (1) subtractive concepts or (2) method of instruction or (3) use of materials cannot, however, be ascertained from the design of the study.

What is the most effective method of teaching pupils to estimate quotient digits?

Meaningful algorithms ultimately may need to be shortened to gain efficiency in division. Then pupils must be able to estimate quotient digits systematically. Several methods have been advocated: (1) the "apparent" or "round-down" method, in which the divisor is rounded to the next lower multiple of 10; and (2) the "increase-by-one" methods, in which the divisor is rounded to the next higher multiple of 10, (a) either "round-both-ways," depending on whether the digit in units' place is less or greater than 5, or (b) "round-up," no matter what. Which method do you use?

	apparent or round-down	increase-by-one	
		round-up	round-both ways
$42\overline{)216}$	$4\overline{)21}\,5$	$5\overline{)21}\,4$	$4\overline{)21}\,5$
$47\overline{)216}$	$4\overline{)21}\,5$	$5\overline{)21}\,4$	$5\overline{)21}\,4$

Efforts to resolve the issue of which method is best have focused on analysis and comparison of the success of each

method on a specified population of division examples. Morton (1947), for instance, analyzed 40,014 examples and found that an "increase-by-one" method was "correct" 61% of the time and the "apparent" method was "correct" in 53% of the cases. Grossnickle conducted a series of such studies, as did Osburn (1950), carefully comparing the cases where each method resulted in the "correct" quotient digit.

If success on first trial were the criterion, then "round-both-ways" would be recommended. However, not only must the child learn two rules, but the true quotient digit may then be either greater or less than an estimated quotient digit. Grossnickle (1932) and Osburn (1946) advocated the "apparent" method, since the estimated digit is always corrected (if necessary) by decreasing it. With the "round-up" method, the estimated digit is corrected (if necessary) by increasing it -- a procedure very much like that used in the subtractive algorithm.

Hartung (1957) critically reviewed these and other analytic studies. He concluded that "round-up" was the most useful method, because of the advantages of obtaining an estimate that is less than the true quotient (which decreased the need for erasing), and because of the relative simplicity of a "one-rule" method.

In one of the few experimental investigations on this topic, Grossnickle (1937) studied the achievement of groups taught by "round-down" and "round-both-ways." He concluded that there were no significant differences between the scores of the two groups.

How children apply the method was studied by Flournoy (1959), who found that "round-both-ways" was used as effectively as the "round-down" method. She stressed that perhaps not all children should be taught the "round-both-ways" method. Carter (1960) reported that pupils taught this method were not as accurate as those taught a one-rule method -- nor did pupils always use the method taught.

What is the role of measurement and partition situations in teaching division?

Measurement problems involve situations such as:

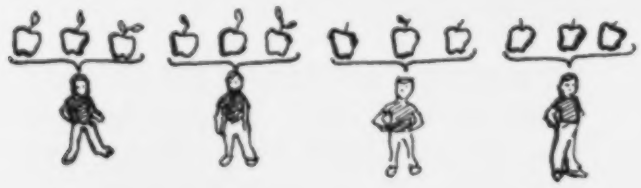
> If each boy is to receive 3 apples, how many boys can share 12 apples? (Find the number of equivalent subsets.)

Partition problems involve situations like this:

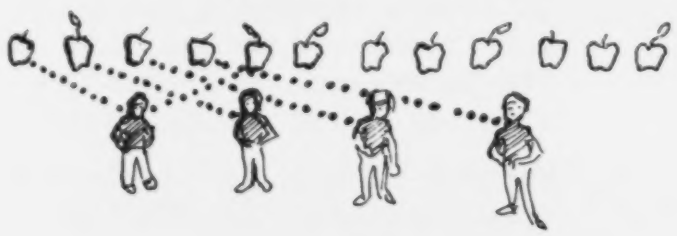
> If there are 4 boys to share 12 apples equally, how many will each boy receive? (Find the number of elements in each equivalent subset.)

In a study with second graders (chosen since commonly children at this level have had little experience with division which would interact with the teaching in the

research study), Gunderson (1953) reported that problems involving partition situations were more difficult than problems involving measurement situations. The ease of visualizing the measurement situation probably contributes to this. For instance, for the illustration above, a picture like this could be formed:



For the partition situation, the drawing might be:



and so on!

Zweng (1964) also found that partition problems were significantly more difficult for second graders than measurement problems. She further reported that problems in which two sets of tangible objects were specified, were easier than those in which only one set of tangible objects was specified. In an earlier study, Hill (1952) found that pupils in the intermediate grades indicated a preference for measurement situations, but performance was similar on both types.

In the study in which they compared two division algorithms, Van Engen and Gibb (1956) found that children who used the distributive algorithm had greater success with partition situations, while those who used the subtractive algorithm had greater success with measurement situations.

Scott (1963) used the subtractive algorithm for measurement situations and the distributive algorithm for partition situations. He suggested that: (1) use of the two algorithms was not too difficult for third grade children; (2) two algorithms demanded no more teaching time than only one algorithm; and (3) children taught both algorithms had a greater understanding of division.

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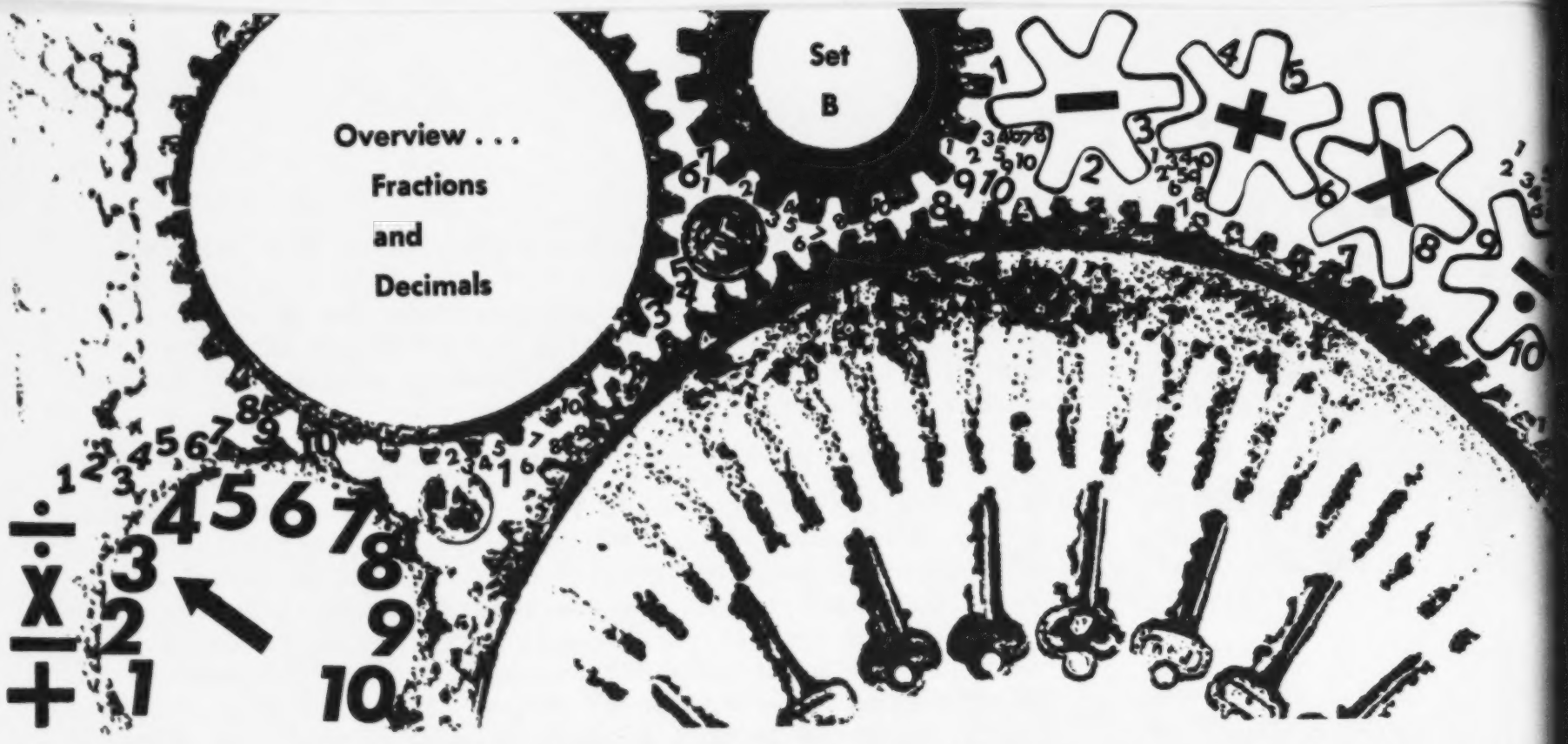

RATIONAL NUMBERS: FRACTIONS AND DECIMALS

Can young children learn fractional concepts?

We know that children come to school with some knowledge about fractions: at least 50 per cent can recognize halves, fourths, and thirds. They can extend this knowledge beginning in the primary grades, especially with a systematic program emphasizing the use of manipulative materials.

How should children find the common denominator for addition and subtraction with fractions?

The little research evidence on this question indicates that the procedures of (1) setting up rows of equivalent fractions and (2) factoring the denominators are both effective. That most errors are made by pupils when "reducing," when determining the numerator, and when adding needs to be considered as we plan lessons. We should also devote particular attention to examples in which pupils have the most difficulty, those in which the common denominator is not apparent.

Is it helpful to analyze errors pupils make with fractions?

In general, for all processes with fractions, we know that errors are most frequently caused by (1) difficulty with "reducing," (2) lack of comprehension of the process, and (3) computation. If we plan carefully to help pupils identify and correct their errors, greater achievement, with accuracy, should result.

Greater attention to regrouping and to "cancellation" might also help pupils to avoid errors when these two procedures are needed.

How can we most effectively rationalize the algorithm for multiplication with fractions?	There is little research evidence to answer this question. We know that for multiplication with fractions (as for other operations), use of programmed materials and of multi-level materials are effective. Using the inversion method to teach division of fractions may also increase achievement in multiplication with fractions.
What algorithm shall we use for division with fractions?	Most studies have indicated that use of either the inversion or the reciprocal algorithm is probably most effective for most types of examples requiring division with fractions. When pupils are taught why the inversion algorithm works (by using the reciprocal principle), retention seems to be improved. You might consider using the common denominator algorithm as an alternate procedure for pupils having difficulty, since it is most closely related to division with whole numbers.
What other things contribute to improved achievement with fractions?	Teaching about fractions and operations with fractions meaningfully has been found to be effective. Having pupils manipulate materials and providing practice are also helpful, of course.
Is it helpful to relate decimals with fractions or place value?	You should apparently place emphasis on both fractions and place value: when decimals are taught only in relation to place value, achievement and retention are not as high as when emphasis is placed on both numeration and the relationship to fractions. There is some evidence to suggest, however, that since computation with decimals seems to be more nearly like computation with whole numbers than like computation with fractions, reinforcement of whole number computational skills is provided when decimals are taught before fractions.
How should we teach children to place the decimal point in division with decimals?	Research indicates that to facilitate understanding we should teach children to locate the decimal point in the quotient by making the divisor a whole number by multiplying it by a power of 10, and then multiplying the dividend by the same number. Greater accuracy results than when children merely subtract the number of decimal places in the divisor from the number of places in the quotient.

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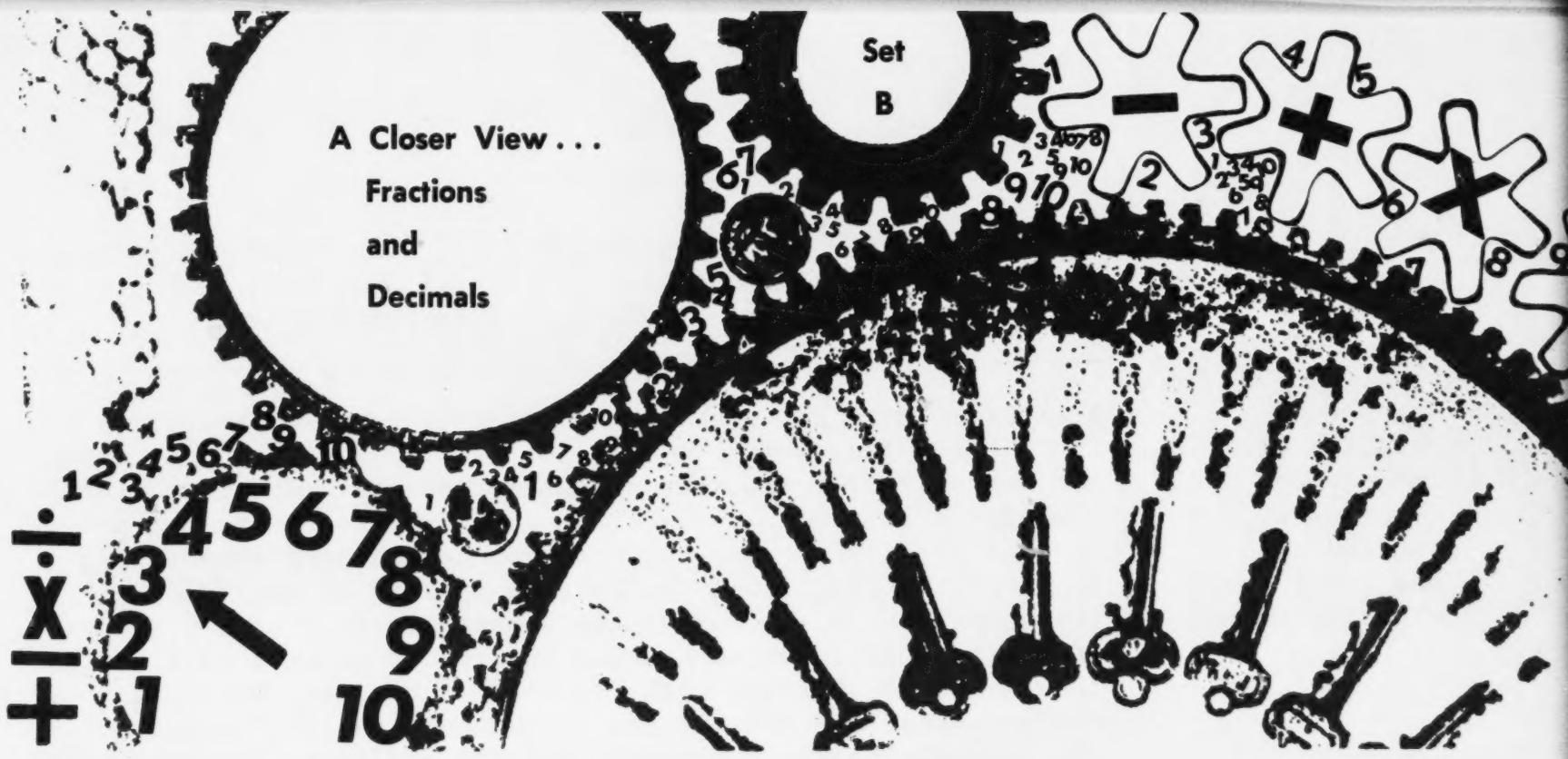

RATIONAL NUMBERS: FRACTIONS AND DECIMALS

Since several interpretations of the above words are possible, let's clarify how we're using them. We shall use the word <u>fraction</u> to refer to a <u>number</u>: a number that may be expressed in the form $\frac{a}{b}$, where <u>a</u> and <u>b</u> are whole numbers and $b \neq 0$. The word <u>decimal</u> will be used to refer to a particular kind of fraction: one that is expressed in our familiar positional place-value notation, with the denominator being some power of 10.

Can young children learn fractional concepts?

We have found from surveys of what children know about mathematics upon entering school that at least 50 per cent can recognize halves, fourths, and thirds, and have acquired some facility in using these fractions. Gunderson and Gunderson (1957) interviewed 22 second graders following their initial experience with a lesson on fractional parts of circles. The investigators concluded that fractions could be introduced at this grade level, with the use of manipulative materials and through oral work with no symbols used.

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It should be noted that research is variable with respect to its quality; hence, the same degree of confidence cannot be placed in all findings. An attempt has been made to take this fact into consideration in preparing this bulletin.

A planned, systematic program for developing fractional ideas seems essential as readiness for work with symbols. Use of manipulative materials is vital in this preparation.

How should children find the common denominator for addition and subtraction with fractions?

There is little evidence on the effectiveness of procedures for finding the common denominator in addition with fractions, and even less for subtraction with fractions. Anderson (1966) analyzed errors made by 26 fifth grade classes using two procedures for finding the least common denominator when adding two "unlike" fractions: by setting up rows of equivalent fractions, and by factoring the denominators. There were no significant differences between the two procedures on tests of four kinds of addition with fractions examples. Furthermore, Anderson reported that errors connected with (1) "reducing," (2) determining the numerator, and (3) addition, occurred most frequently, with the greatest frequency of error in examples in which the least common denominator was not apparent.

Bat-haee (1969) compared 112 fifth graders who were taught (1) the factoring method or (2) the "inspection" method of a current textbook series. Those taught by the factoring method scored significantly higher on the experimental posttests.

Is it helpful to analyze errors pupils make with fractions?

Many earlier studies were concerned primarily with the specific errors children make. In general, it was found that, for all operations with fractions, the major errors were caused by (1) difficulty with "reducing," (2) lack of comprehension of the operation involved, and (3) computational errors (e.g., Brueckner, 1928a; Morton, 1924; Schane, 1938). Such findings frequently influenced the material included in textbooks.

Guiler (1936) was among those who reported success with a remedial program which provided practice on correcting errors which had been identified. Ramharter and Johnson (1949) had "good" and "poor" achievers think aloud while they attempted to correct errors in six examples involving subtraction with fractions. On subsequent tests, "good" achievers consistently corrected more errors, using a guidesheet effectively.

Aftreth (1958) had sixth grade pupils identify and correct errors imbedded in 19 completed sets of examples in addition and subtraction with fractions, while a control group worked the examples. No significant differences on either immediate or delayed recall tests were found for addition with fractions, while some significant differences favoring the group working the examples were found for subtraction with fractions. The author suggested that having pupils correct their own errors might be more effective than having them correct imbedded errors.

Fifth graders tested by Scott (1962) made more errors in subtraction with fractions involving regrouping than in subtraction with whole numbers involving regrouping. He suggested that current emphasis on the decimal system may reduce the "flexibility" which the child must have to deal successfully with subtraction with fractions when regrouping is necessary.

Romberg (1968) reported that among sixth graders who used a correct algorithm to multiply fractions, about twice as many pupils in "modern" programs as in "traditional" programs either did not express products in simplest form (as directed) or made errors in doing so. He attributed this difference to pupils' failure to "cancel," and suggested that the cancellation process is important -- even essential -- if <u>efficiency</u> in multiplication is one of the desired outcomes of instruction.

How can we most effectively develop the algorithm for multiplication with fractions?

There is little research evidence to answer this question. Recent research on multiplication with fractions has been primarily within the context of programmed instruction, where the purpose of the investigation was to compare various programming strategies, while fractions served merely as the content vehicle. For instance, Kyte and Fornwalt (1967) used programmed materials on multiplication with fractions to ascertain the rate of mastery by pupils at two IQ levels. While they found that pupils with superior IQ's were able to master identified types of examples more quickly than those with normal IQ's, the study says nothing about what procedures they used to teach the operation with fractions.

Miller (1964) found that significantly higher gains in multiplication with fractions were made by pupils using programmed practice materials, which provided immediate knowledge of answers, than by pupils using conventional textbook materials. In another investigation, higher achievement on the experimental posttest resulted when multiplication with fractions was taught with multi-level materials rather than with single textbooks (Triplett, 1963).

What algorithm shall we use for division with fractions?

Bergen (1966) prepared booklets designed to teach pupils by complex fraction, common denominator, or inversion algorithms. No significant differences were found between complex fraction and inversion algorithms, but each was significantly superior to the common denominator algorithm on most types of examples.

common denominator

$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \div \frac{2}{4} =$

$\frac{3 \div 2}{4 \div 4} = \frac{3 \div 2}{1} = 3 \div 2 =$

$\frac{3}{2}$

Sluser (1963) compared teaching the common denominator and inversion algorithms with and without explanation of the reciprocal principle as the rationale behind inversion. The group given the explanation scored lower on tests of division with fractions than a group merely taught to invert and multiply. He suggested that only above average pupils could

inversion

$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} =$

$\frac{6}{4} = \frac{3}{2}$

understand the principle. However, a large percentage of errors occurred because pupils performed the wrong operation. Krich (1964) reported no significant differences on immediate posttests for pupils taught why the inversion procedure works, as compared with those merely taught the rule. On retention tests requiring recall, however, the group taught with meaning scored significantly higher.

complex fraction

$\frac{3}{4} \div \frac{1}{2} = \frac{\frac{3}{4}}{\frac{1}{2}} =$

$\frac{\frac{3}{4} \times \frac{2}{1}}{\frac{1}{2} \times \frac{2}{1}} = \frac{\frac{6}{4}}{1} =$

$\frac{6}{4} = \frac{3}{2}$

In a study by Capps (1963) the effectiveness of the common denominator and inversion algorithms for division with fractions was compared. There were no significant differences in achievement on tests of addition, subtraction, and division with fractions, while pupils taught the inversion algorithm scored significantly higher on immediate posttests and on retention tests of multiplication with fractions than those taught the common denominator algorithm. This retroactive effect on multiplication was also reported by Bidwell (1968). He found that the inverse operation procedure was most effective, followed by complex fraction and common denominator procedures. The complex fraction procedure was better for retention, while the common denominator procedure was poorest.

What other things contribute to improved achievement with fractions?

Howard (1950) reported on a study with 15 classes of pupils in grades 5 and 6 who were taught addition of fractions by three methods differing in the amount of emphasis on meaning, use of materials, and practice. Pupils retained better when they learned fractional work through extensive use of materials and with considerable emphasis on meaning, plus provision for practice. Krich (1964), Shuster and Pigge (1965), Sebold (1946), and Feinstein (1952) also support the importance of using meaningful methods for work with fractions.

Many other investigations have been done in which fractions have served as the content. For example, Fincher and Fillmer (1965) were interested in exploring programmed instruction variables. They reported that programmed materials were more effective in teaching addition and subtraction with fractions than was conventional classroom instruction.

Is it helpful to relate decimals with fractions or place value?

Faires (1963) introduced some pupils to decimals through a sequence based on an orderly extension of place value, with no reference to common fraction equivalents, while others were taught fractions before decimals, as is usually done. Gains in computational achievement and at least as good an understanding of fraction concepts resulted. Faires indicated that "computation with decimals is [apparently] more nearly like computation with whole numbers than with fractions;" thus reinforcement of whole number computational skills is provided.

O'Brien (1968) reported that pupils taught decimals with an emphasis on the principles of numeration, with no mention of

fractions, scored lower on tests of computation with decimals than those taught either (a) the relation between decimals and fractions, with secondary emphasis on principles of numeration, or (b) rules, with no mention of fractions or principles of numeration. On later retention measures, the numeration approach was significantly lower than use of the rules approach, but not significantly different from the fraction-numeration approach.

How should we teach children to place the decimal point in division with decimals?

Brueckner (1928b) and Grossnickle (1941) analyzed the difficulties with decimals which children have, citing misplacing of the decimal point in division as one of the major sources of error. Flournoy (1959) compared sixth grade classes taught to locate the decimal point in the quotient by (1) making the divisor a whole number by multiplying by a power of 10, and then multiplying the dividend by the same number, or (2) subtracting the number of decimal places in the divisor from the number of places in the dividend. Multiplying by a power of 10 resulted in greater accuracy, as Grossnickle (1941) had concluded earlier.

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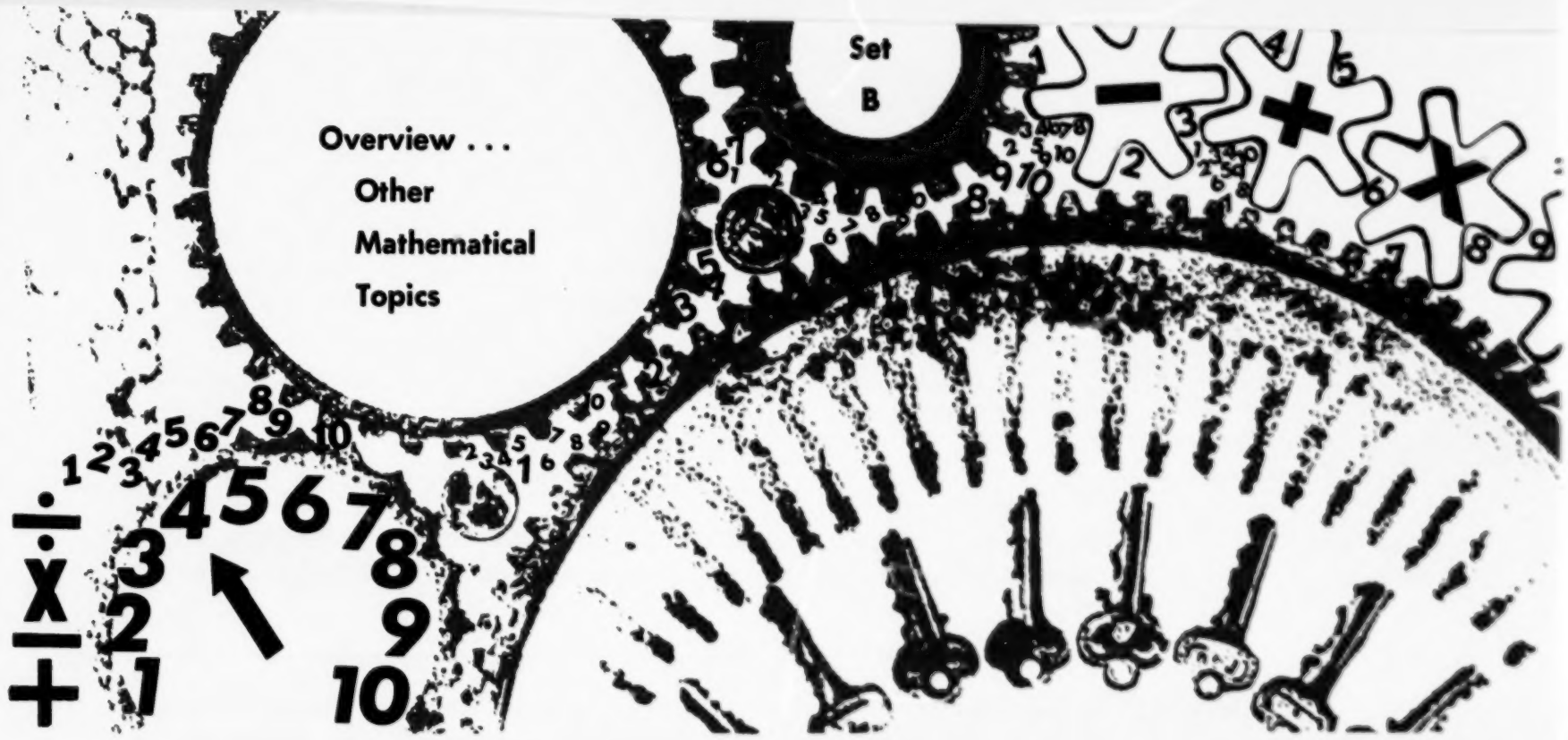

OTHER MATHEMATICAL TOPICS

What measurement and geometry is included in textbooks and programs?

Beginning in most third grade textbooks, measurement content is organized by units, with emphasis on relationships among standard units developed by grade 6. Few experiences in creating measures, applying measuring ideas, and actually measuring were noted. The amount of geometry in the program has increased threefold since 1900, with separation of two- and three-dimensional ideas common.

What do children know about geometry and measurement?

There is evidence that children can learn many geometric ideas associated with plane figures. They can learn to make simple constructions, though lack of precision in using the compass results in many errors.

Wide differences in familiarity with measurement ideas are evident. It has been suggested that (1) some ideas now taught in first grade are probably already part of the child's knowledge when he enters school, and (2) teachers need to take into account the age, socioeconomic level, and mental ability when planning measurement activities.

How can we help pupils understand our numeration system?

There is some evidence that learning about other bases increases understanding of the decimal numeration system. However, emphasizing the structure and properties of the decimal system seems just as effective.

What effect does the teaching of properties and relations have?	Teaching the commutative, associative, and distributive properties and various relations may facilitate other mathematical learning, but research on this is limited.
What can pupils learn about . . .	
integers?	The little research evidence on this topic indicates only that concrete and abstract approaches may each be effective.
. . . set concepts?	Ideas about sets appear to be useful in introducing both numerical and geometric concepts. A teaching sequence using (1) physical action, (2) manipulation of concrete materials, and (3) observation of semi-concrete illustrations seemed effective in teaching about sets. Several studies have suggested that pictures of objects and groupings should be kept relatively simple.
. . . probability and statistics?	Intermediate grade children apparently have acquired considerable familiarity with probability from everyday experiences, and can apply knowledge about finite sample spaces and the probability of certain events occurring. The mode, the mean, and possibly the median can be introduced as early as grade 4.
. . . logic?	Children aged 6 through 8 may be able to recognize valid conclusions derived from sets of given premises, though they may have difficulty testing the logical necessity of a conclusion.

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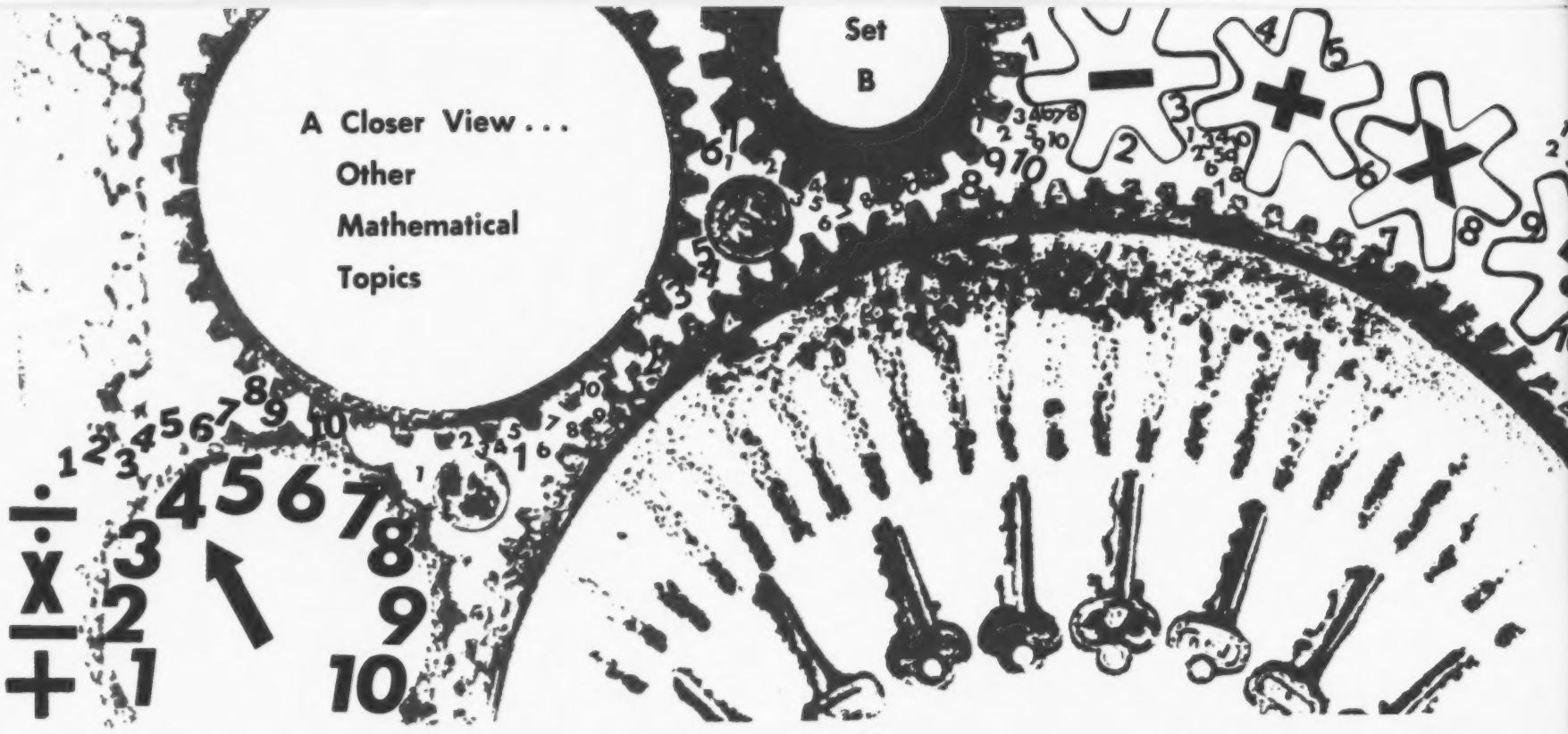

OTHER MATHEMATICAL TOPICS

What measurement is included in elementary school textbooks?

Paige and Jennings (1967) surveyed 39 textbook series, summarizing the measurement content. Starting in third grade, about half of the books put measurement concepts in a separate chapter. In most fourth grade books, problems generally involved regrouping with measures and conversions. By grade 5 most series had developed the ideas of standard units and errors in measuring. Other relationships between measures were introduced in many series in grade 6. Paige and Jennings noted that there were few experiences where students created their own units of measure, too little emphasis on practical application, and too few problems requiring actual measuring.

Is there common agreement on what geometry will be presented?

Neatrour (1969) analyzed 16 textbook series and surveyed 156 middle schools to determine the status of geometric content in their curricula. He found that while the amount of geometric content varied greatly, three times as much was included as in 1900, with emphasis on informal geometry. Compartmentalization of geometric content into two- and three-dimensional ideas was common.

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What geometric ideas can children learn?

From a set of tests administered after two weeks of teaching, Shah (1969) reported that children aged 7 to 11 learned concepts associated with plane figures, nets of figures, symmetry, reflection, rotation, translation, bending and stretching, and networks. In a pilot study, Denmark and Kalin (1964) found that fifth graders could satisfactorily (1) bisect an angle, (2) construct the perpendicular bisector of a line segment, (3) copy a triangle, (4) construct a perpendicular to a line through a point on the line, and (5) copy a quadrilateral. Lack of precision in the use of the compass accounted for many errors.

D'Augustine (1966) used programmed texts on topics such as paths and their properties, simple closed curves, and polygons with pupils in grades 5, 6, and 7. He reported that reading and mathematics achievement significantly affected success, but age, length of class period, grade, or sex did not.

What do children know about measurement?

What can they learn about measurement?

Four- and five-year-olds exhibit wide differences in familiarity with ideas of time, linear and liquid measures, and money, with little mastery evident (Davis, Carper, and Crigler, 1959). In another survey with first graders, Mascho (1961) reported that as age, socioeconomic level, or mental ability increased, the children's familiarity with measurement increased. Familiarity was greater when the terms were used in context. It was suggested that (1) some ideas now considered appropriate for first grade should be considered part of the child's knowledge when he enters school, and (2) teachers need to study the composition of their groups in terms of age, socioeconomic level, and mental ability when planning curricular activities with measurement. This may be especially important in view of Piaget's findings, which suggest that general concepts of linear measurement are not attainable for children until approximately age 8, when the child appreciates that a linear segment may be conserved even when subdivided.

Friebel (1967) found seventh graders using SMSG materials were significantly superior to those using "traditional" materials in understanding of and skill in using measurement concepts. However, "in process estimation of the measures of common quantities," both groups were equally adept except when dealing with area and volume, where the SMSG students were better.

Corle (1960) substantiated the need for experiences with measurement. He found that sixth graders could estimate weight, size, temperature, and time more accurately than fifth graders, but error was 45% for sixth grade and 61% for fifth grade. Sixth grade pupils measured with acceptable accuracy only about half the time; fifth graders, one-third of the time.

What aspects of graphing can be learned?

Dutton and Riggs (1969) used a programmed text to present pictographs and circle, bar, and line graphs to 393 fourth and fifth graders. The text was effective in improving skills on both a graph test and on graph interpretation items from a standardized test. There is some evidence from other research that, for third graders, pictographs and bar graphs are easier to interpret than line graphs.

How can we help pupils understand our numeration system?

Flournoy, Brandt, and McGregor (1963) found that the items missed very frequently by pupils in grades 4-7 on tests measuring understanding of our numeration system related to: (a) the additive principle; (b) making "relative" interpretations; (c) meaning of 1000 as 100 tens or 10 hundreds, etc.; (d) expressing powers of ten, as 1000 = 10 x 10 x 10; and (3) the 10-1 place value relationship. Thus greater emphasis on these is necessary as we teach.

The study of non-decimal numeration systems was included in many modern mathematics programs because it was presumed that such work would strengthen understanding of the decimal numeration system. There is some evidence that kindergarteners, first graders, and fourth graders showed an increase in their understanding of the decimal system after a study of another base. Jackson (1965) concluded that fifth graders taught non-decimal systems did significantly better than pupils taught only the decimal system, on tests measuring understanding of the decimal system, properties, and problem solving. Those receiving instruction only in the decimal system did significantly better on computation in that system.

On the other hand, Scrivens (1968) concluded that study of non-decimal numeration systems is "inappropriate" for third graders and Schlingsog (1968) reported no significant differences on tests of understanding and computation in base ten between groups who were taught about other systems and those who studied only the decimal system. Kavett (1969) reported similar results for the reasoning scores of fourth and sixth graders, though retention scores were significantly higher for the groups taught non-decimal numeration. Smith (1968) found that study of non-decimal numeration systems by fourth graders produced a greater understanding of non-decimal systems but not of the decimal system.

What effect does the teaching of properties and relations have?

We believe that learning about properties will facilitate understanding, but research on this is very limited. Schmidt (1966) reported that teaching the commutative, associative, and distributive properties significantly increased fourth graders' ability to apply the fundamental processes to examples and problems. Sixth graders learned a significant amount about topics such as the reflexive, symmetric and transitive properties of some relations, equivalence relations, and graphing relations, but no significant difference was found in their ability to perform on traditional problems (Gravel, 1968).

Other researchers have reported that the properties may be too difficult for second and fourth graders to understand, and that seventh graders apply properties better than fifth graders.

What can pupils learn about integers?

There has been almost no research which provides an answer to this question. An exploratory study with six primary grade children showed that they could be taught some concepts about integers when the number line is used. Coltharp (1969) reported no significant difference in achievement between sixth graders taught addition and subtraction of integers from an abstract, algebraic approach and those taught by means of a concrete, visual approach. According to Tremel (1964), success in learning

to add and multiply integers was not related to numerical and spatial abilities, but was related to verbal and problem solving abilities.

What set concepts facilitate achievement?

This is another example of a topic which has influenced modern programs tremendously, yet evidence is woefully lacking. It is generally accepted that many of the elementary terms and operations of set theory are useful and desirable in the elementary mathematics program. In fact, the ideas of "sets" are unavoidable in the introduction of number concepts and intuitive geometry, though the formal terms may not be used.

There has been some concern with how to picture groups of objects. In two older studies, Carper (1942) and Dawson (1953) concluded that the greater the complexity of the objects and the group configuration, the greater the difficulty children have in determining how many are in the group. Thus in the primary grades it seems important to picture relatively simple objects and groupings.

Suppes and McKnight (1961) found that concepts and operations with sets could be taught in grade 1, noting that "operations on sets are more meaningful to the student than operations on numbers," since sets are concrete objects. As long as the notation introduced is explicit and precise and corresponds to simple concepts, no difficulties of comprehension seemed to arise. Holmes (1963), however, reported that first graders scored below the 50% level for tests on equality concepts, ordinal number, subsets, and number property of sets.

Harper, Steffe, and Van Engen (1969) reported success in teaching conservation of numerousness, including one-to-one correspondence and equivalent and non-equivalent sets, to children at the first grade level. They noted that "the teaching sequence used in these lessons, i.e., a progression from physical action of the children, to their manipulation of concrete materials, to their observation of semi-concrete illustrations, seems to be an effective approach to use in teaching early number concepts." [*Underlining added.*]

What can children learn about probability and statistics?

Intermediate grade children apparently have acquired considerable familiarity with probability from everyday experiences, and can apply knowledge about (1) a finite sample space, (2) the probability of a simple event in a sample space, (3) the probability of the union of non-overlapping events, (4) the difference between mutually independent and mutually exclusive events, and (5) quantification of probabilities (Doherty, 1966; Leffin, 1969).

Smith (1966) concluded that the following topics of probability and statistics seem to be appropriate for most seventh grade students: (1) possible outcomes of an experiment, (2) probability of events that are equally likely and events that are not equally likely, (3) mutually exclusive events, (4) Pascal's triangle, (5) histograms, (6) continuous and discrete data, (7) central tendency, and (8) measures of variation. There is some evidence from another study that the mode, the mean, and possibly the median can be introduced as early as grade 4.

What can children learn about logic?

If the child is to learn to think critically, it is important that he make logically correct inferences, recognize fallacies, and identify inconsistencies among statements. Hill (1961) concluded that children aged 6 through 8 are able to recognize valid conclusions derived from sets of given premises. There seems to be a "gradual, steady growth which is nearly uniform for all types of formal logic." Differences in difficulty were associated with type of inference, but these difficulties were specific to age. Difficulties associated with sex were not significant. Children can learn to recognize identical logical form in differing content. The addition of negation very significantly increased difficulty in recognizing validity. Roberge (1969) reported that negation in the major premise also had a marked influence on the development of logical ability in children in grades 4, 6, 8, and 10.

O'Brien and Shapiro (1968) confirmed Hill's findings, except that "little growth was detected between ages 7 and 8." Using a modification of Hill's test, they found that children experienced great difficulty in testing the logical necessity of a conclusion, and showed slow growth in this ability, which supports Piaget's theory that children reach the stage of ability to think logically later than age 8. They caution that Hill's research should be interpreted and applied with caution: hypothetical-deductive ability cannot be taken for granted in children of this age.

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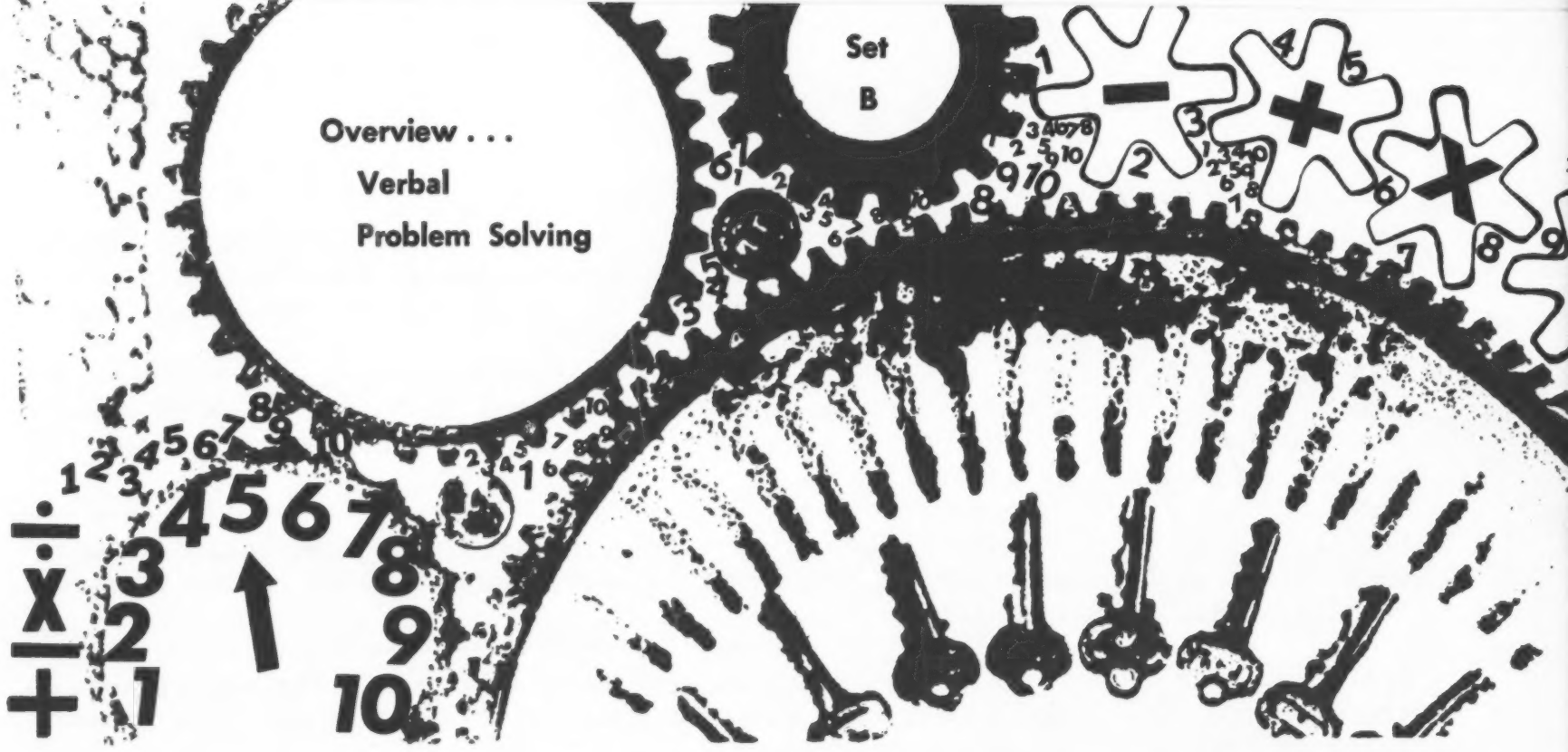


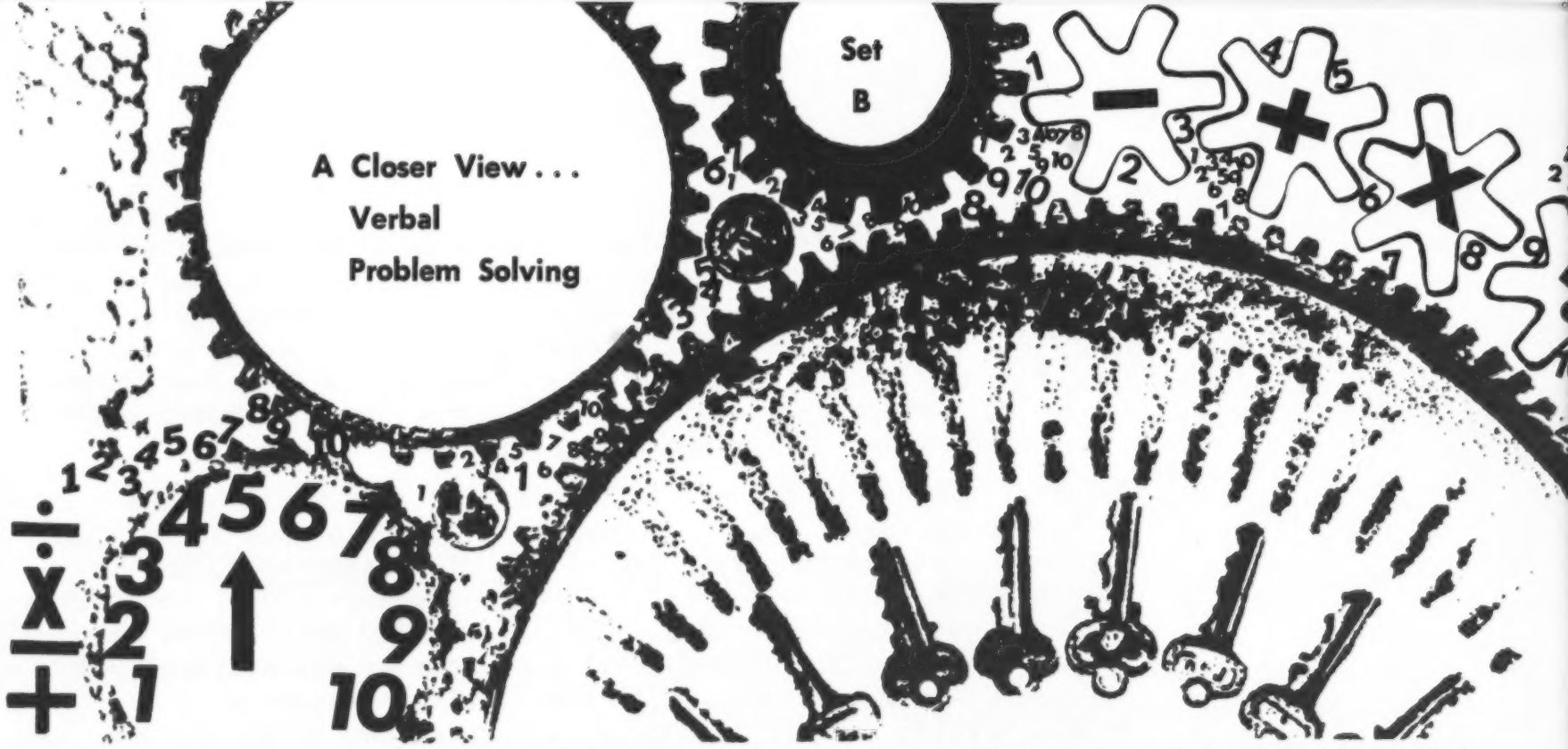
VERBAL PROBLEM SOLVING

What factors are related to problem solving ability?	Intelligence is related to problem solving ability; however, neither sex nor socio-economic status has been found to be related to it.
What are the characteristics of good problem solvers?	Among the factors which characterize high achievers are: ability to note likenesses, differences, and analogies; understanding of mathematical terms and concepts; ability to visualize and interpret quantitative facts and relationships; skill in computation; ability to select correct procedures and data; and comprehension in reading. Reasons for difficulty with problem solving generally focus on computation, reading, and knowledge of fundamental mathematical concepts.
How important is reading to problem solving ability?	Reading is obviously important, since if the child cannot read the problem, he will have difficulty in doing more than guessing how to solve it. It is suggested that reading and other interpretive skills specifically related to problem solving be developed in the problem solving program.
What is the role of "understanding?"	Systematic instruction not only in how to solve a problem but in why that process is appropriate has been found to be effective in increasing problem solving achievement and understanding.
Is the study of vocabulary helpful?	Since knowledge of vocabulary has been found to be important to success in problem solving, it follows that instruction in the vocabulary to be used will increase scores.

What problem settings are most effective?	Evidence on whether settings should be familiar to the child is conflicting. It is apparently not as important as has sometimes been supposed: the child will be interested in a variety in settings.
Does the order of processes affect problem difficulty?	There is some evidence to show that the order in which the processes are presented in multi-step problems may affect their difficulty.
Does the order of data affect problem difficulty?	Significantly higher scores resulted when numerical data were presented in the order in which they would be needed to solve the problem.
Should we place the question first or last?	For some children, it appears that a problem is easier when the question is placed first. This shortens the time needed to solve the problem.
What is the role of formal analysis?	Giving children many opportunities to solve problems and letting children solve problems in a variety of ways appears to be more helpful than formal analysis procedures.
What techniques help in improving pupils' ability to solve problems?	While research evidence supporting each is somewhat limited, researchers have suggested that these techniques should be included in the problem solving program: (1) Provide problems at varying levels of difficulty. (2) Have pupils write mathematical sentences. (3) Have pupils dramatize problem situations. (4) Have pupils make drawings and diagrams. (5) Have pupils formulate problems. (6) Present problems orally. (7) Use problems without numbers. (8) Have pupils designate the process to be used. (9) Have pupils note missing or extra data. (10) Have pupils test the reasonableness of their answers. (11) Use a tape recorder to aid poor readers.
Is it helpful for pupils to work in groups?	The evidence suggests that pupils achieve at least as much by working independently when solving problems as by working in groups of two, three, or four.

The material included in this bulletin is a product of the "Interpretive Study of Research and Development in Elementary School Mathematics" (Grant No. OEG-0-9-480586-1352(010), sponsored by the Research Utilization Branch, Bureau of Research, U.S. Office of Education, and conducted at The Pennsylvania State University.

If you would like more information about the research whose findings are cited above, contact MARILYN N. SUYDAM, Project Director, at The Pennsylvania State University, University Park, Pennsylvania, 16802.



VERBAL PROBLEM SOLVING

Verbal problem solving has attracted more attention from researchers than any other topic in the mathematics curriculum. It is considered a plausible way to help children learn how to apply mathematical ideas and skills to the solving of real-life problems--and is a challenge to both pupils and teachers.

It should be noted that virtually all of the research on problem solving has been associated with whole numbers. We lack evidence about the extent to which the research can be generalized to other kinds of numbers. This is a topic for future research.

What factors are related to problem solving ability?

It is generally concluded that:
(1) IQ is significantly related to problem solving ability;
(2) sex differences do not appear to exist in the ability to solve verbal problems; and
(3) socio-economic status alone does not appear to be a significant factor.

What are the characteristics of good problem solvers?

Many researchers have proceeded on the assumption that if we can ascertain what problem solvers who are successful have in common, we may be able to help those who do not do as well. Alexander (1960) and Hansen (1944) compared pupils on selected factors

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The bulletin was prepared by MARILYN N. SUYDAM, The Pennsylvania State University, Project Director, and J. FRED WEAVER, The University of Wisconsin-Madison, Project Consultant. Art by Ed Saffell.

It should be noted that research is variable with respect to its quality; hence, the same degree of confidence cannot be placed in all findings. An attempt has been made to take this fact into consideration in preparing this bulletin.

thought to be related to problem solving ability. Among the factors which characterized high achievers were: (1) ability to note likenesses, differences, and analogies; (2) understanding of mathematical terms and concepts; (3) ability to visualize and interpret quantitative facts and relationships; (4) skill in computation; (5) ability to select correct procedures and data; and (6) comprehension of reading materials.

Related to these findings are the specific errors which John (1930) found that children in grades 4, 5, and 6 made in solving problems: errors in reasoning, in use of fundamentals, and in reading were found to be most frequent. Johnson (1944) noted that other researchers reported similar reasons why children do not succeed in solving problems: (1) ignorance of mathematical principles, rules or processes; (2) insufficient mastery of computational skills; and (3) inadequate understanding of vocabulary. In a more recent study, Chase (1960) reported test data collected from sixth graders showing that the three primary factors related to success in problem solving are computation, reading to note details, and knowledge of fundamental mathematical concepts.

How important is reading to problem solving ability?

Treacy (1944) and Alexander (1960) found that good and poor achievers in problem solving differed on many aspects of reading. Treacy concluded that reading should be regarded as a composite of specific skills rather than as a generalized ability. We may infer that reading and other interpretive skills should be specifically developed in the problem solving program.

Balow (1964) studied 468 sixth graders who had been classified by reading and computational levels. He reported that higher levels of problem solving ability were associated with higher levels of reading and computational ability, but that much of this relationship apparently was the result of the high correlation of these abilities with IQ.

What is the role of "understanding" in problem solving?

We know that many children have difficulty in deciding what process to use to solve a given problem. It therefore has seemed evident to researchers that to make this decision without guessing or using trial and error procedures, pupils must understand both the meanings and the effects of the fundamental processes. Pace (1961) presented one group of fourth graders with systematic instruction in which children not only decided how to solve a problem, but why that process was appropriate, while another group merely solved the problems with no discussion. The first group made statistically significant gains on tests of problem solving. Interviews and other tests used to measure understanding showed that both groups improved, with greater gains for those who received specific instruction.

Is the study of vocabulary helpful in improving problem solving?

Among those who experimented with the teaching of vocabulary was VanderLinde (1964), who reported that such specific instruction on quantitative vocabulary was effective in increasing problem solving scores (for problems in which that vocabulary was used).

What problem settings are most effective?

Whether children's success in solving problems is affected by the familiarity in the settings was studied by many. Brownell and Stretch (1931) reported on the reactions of 256 fifth graders to carefully matched problems at four degrees of familiarity. They concluded that there is "no ground for reasonable belief that problems are made unduly difficult for children by being given unfamiliar settings."

While some other researchers confirmed this finding, there is conflicting evidence on this question. Washburne and Osborne (1926) concluded that unfamiliarity of setting has some influence on success in problem solving, although it is "not as large an element as might be supposed." On the other hand, Sutherland (1942) was among those who found that pupils were decidedly more successful on problems with familiar settings.

It has been concluded by many researchers that children like a variety of problem settings. It seems important that children be interested in problems and in ways of solving them.

In studying a different aspect related to this question, Scott and Lighthall (1967) reported that no statistically significant relationship was found between "need content" in problems and degree of "disadvantage." ("Need content" was defined <u>low</u> if problems concerned food and shelter, and <u>high</u> if they concerned such factors as belongingness, education, travel, etc. "Disadvantage" was determined by whether or not pupils were assured of food and shelter.)

Does the order in which fundamental processes appear affect problem difficulty?

Citing data from 4,444 pupils in grades 4, 6, and 7, Berglund-Gray and Young (1940) said "yes." They reported that the easier order for each pair of operations with whole numbers in two-step problems was: addition before subtraction or division; subtraction before division; and multiplication before any of the three others. However, we should note that this study was conducted at a time when there was considered to be only <u>one</u> way of solving a problem.

Does the order of data affect problem difficulty?

Burns and Yonally (1964) reported that, when the data in each of ten multi-step problems were in the order required to solve them, significantly higher scores resulted than when data were not in the order in which it would be used. For the 95 fifth graders they studied, reasoning ability was positively related to pupil success with problems which presented numerical data in mixed order.

Should we place the question at the beginning or the end of a problem?

Williams and McCreight (1965) concluded that for fifth and sixth graders, there was "some advantage to the child when the question was placed first," though no significant difference between mean scores was found. Time to solve was less when the question was placed at the beginning.

What is the role of formal analysis in problem solving?

Research evidence does not show that formal analysis (that is, requiring pupils to answer a specific set of questions in order) is an effective procedure (e.g., Burch, 1953). Washburne and Osborne (1926) noted that "merely giving many problems...appears to be most effective." Pace (1961) also suggested that giving <u>many opportunities</u> to solve problems and letting children solve problems <u>in a variety of ways</u> were especially helpful.

What other techniques help in improving pupils' ability to solve problems?

Many specific techniques have been reported to be helpful, though how helpful has been impossible to determine from the structuring of the research studies. Among the techniques which researchers suggest are:

(1) Provide a differentiated program, with problems at appropriate levels of difficulty.
(2) Have pupils write the number question or mathematical sentence for a problem.
(3) Have pupils dramatize problem situations and their solutions.
(4) Have pupils make drawings and diagrams using them to solve problems or to verify solutions to problems.
(5) Have pupils formulate problems for given conditions.
(6) Present problems orally.
(7) Use problems without numbers.
(8) Have pupils designate the process to be used.
(9) Have pupils note the absence of essential data, or the presence of unnecessary data.
(10) Have pupils test the reasonableness of their answers.
(11) Use a tape recorder to aid poor readers.

Some evidence exists to support each of these. Keil (1965) found that pupils who wrote and solved problems of their own were superior in problem solving ability to pupils who had the "usual textbook experiences." Riedesel (1964) reported that sixth grade classes using specific procedures plus 30 sets of verbal problems at two levels of difficulty achieved higher mean gains on problem solving tests than did control groups who followed the regular textbook program. For instance, Arnold (1969) reported evidence from sixth graders favoring the expression of problem relationships in number sentences. It should be noted that emphasis upon isolated word cues ("left," "in all," etc.) can be grossly misleading as a problem solving procedure. They may lead pupils away from recognition of the relationships inherent in the problem, which are crucial to its solution.

How should equations for problems be stated?

In a well-controlled study, Wilson (1967) studied two problem solving procedures, one using equations which express the real or imagined actions in the problem (an "action-sequence" structure) and the other using equations which emphasize operations by which the problem may be solved directly (a "wanted-given" structure), and a third practice-only control treatment. He reported that differences for ability to choose the correct operation, accuracy, and speed favored those taught the "wanted-given" structure over those taught the "action-sequence" structure on tests given during instruction and after a nine-week retention period. The "wanted-given" structure was also significantly better than the practice-only treatment on the immediate posttest and the retention test. On the other hand, Lindstedt (1963) reported many differences favoring a group who used a text program in which equations are structured in terms of the action, over a group using a "traditional type of problem solving program."

Could it be that one of these procedures is better than the other for certain children?

Is it helpful for pupils to work together in solving problems?

Evidence by investigators in other areas has indicated that children can learn more by working with partners or small groups than by working alone. In relation to verbal problem solving, however, this evidence has not been so clear.

Hudgins (1960) reported that fifth graders who worked on sets of verbal problems in groups of four solved significantly more problems than those who worked alone. When they then worked individually, no significant differences were found among their scores. In an extension of this study, Hudgins and Smith (1966) found that for pupils in groups of three, group solutions to problems were no better than the independent solutions of the most able member of the group, if he is perceived to be most able. (If he is not so perceived, the group will do better than he--or change their perception of him.)

Klugman (1944) found that two children working together at grades 4, 5, and 6 solved more problems correctly, but took a longer time than pupils working alone. In another study with fourth, fifth, and sixth graders, Dembo (1969) reported that there were no significant differences in the improvement of peer relations, attitude toward mathematics, or mathematical achievement between pupils working in small groups or independently.

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Project on Interpreting Mathematics Education Research
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Are you aware of these sources of information on current research in mathematics education?

- **National Council of Teachers of Mathematics (NCTM)**

  -- presents research sessions at name-of-site and national meetings
  -- publishes research reports in The Arithmetic Teacher
  -- publishes the Journal for Research in Mathematics Education
  Information on these and other research publications can be secured from: NCTM
  1201 Sixteenth Street, N.W.
  Washington, D.C. 20036

- **American Educational Research Association (AERA)**

  -- presents research sessions at national meetings
  -- publishes research reports in its journals
  -- sponsors a Special-Interest Group on Research in Mathematics Education
  Information can be secured from: AERA
  1126 Sixteenth Street, N.W.
  Washington, D.C. 20036

- Investigations in Mathematics Education, A Journal of Abstracts and Annotations (School Mathematics Study Group)

  -- presents abstracts and critiques of recent research
  -- lists current research reports and dissertations
  Volumes 1, 2, and 3 may be obtained from: A. C. Vroman, Inc.
  2095 E. Foothill Blvd.
  Pasadena, Calif. 91109

- **Science and Mathematics Education Information Analysis Center, ERIC**

  -- prepares lists of available ERIC materials related to mathematics and mathematics education
  -- publishes proceedings of research sessions at conferences
  For further information, contact: SMAC - ERIC
  The Ohio State University
  1460 West Lane Avenue
  Columbus, Ohio 43221

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Many of you asked for further information on the five films which have been developed for the Project. These films illustrate selected research findings; a brief description of each film follows:

Using a Mathematics Laboratory Approach

While research on the use of mathematics laboratories is limited, results of studies on other topics have been applied in this setting. Use of materials is stressed: ways in which these can be used effectively are depicted. Grouping procedures with emphasis on individualized instruction are relevant.

The intent of the film is not only to acquaint teachers with the laboratory approach, but also to help them in planning and organizing for the use of such an approach. Illustrative activities at several grade levels are shown. Pupil-teacher roles are evident, as the teacher aids individuals and small groups.

The film seeks to answer these questions:

(1) What are mathematics laboratories?

(2) Why use mathematics laboratories?

(3) How is a mathematics laboratory organized?

(4) What are some activities which are valuable in the mathematics laboratory?

Using Diagnosis in a Mathematics Classroom

If mathematics instruction is to be improved, pupil needs must be effectively diagnosed: what is ascertained must be effectively used, with careful planning. Use of interview inventories and a diagnostic instrument are shown.

Grouping pupils on the basis of such evaluation and providing instruction to meet specific needs are explored, through lessons on regrouping in subtraction.

Operations with Whole Numbers

Research evidence on these topics tends to be aimed at specific points. Procedures for interrelating addition and subtraction are explored. Materials, methods and strategies which are particularly effective shown in use.

Attention is focused on the use of multiple techniques for improving instruction in multiplication. Algorithms which have been found to be particularly effective are emphasized. Stress is placed on the use of materials.

Practicing Mathematics Skills

The effectiveness of drill and practice is highly dependent on when it is used and how it is presented. A classroom scene in which it is evident that pupils have a firm understanding is followed by scenes of the teacher presenting appropriate drill-and-practice activities.

Suitable materials, use of a computer-terminal, techniques for promoting interest, and ways of identifying appropriate times for drill are illustrated.

Problem Solving Techniques

While more research is available on this topic than on any other, teachers across all grade levels continue to be perplexed about effective ways to promote verbal problem skills. Examples of teachers and pupils in action, using various problem solving techniques, are interspersed with examples of actual work which results from the use of multiple approaches. The film ranges across content areas, without stressing any particular content.

THE FILMS WILL BE AVAILABLE FROM TWO SOURCES:

(1) AUDIO VISUAL SERVICES, THE PENNSYLVANIA STATE UNIVERSITY, UNIVERSITY PARK, PA. 16802. (AT A MODEST RENTAL CHARGE)

(2) THE NATIONAL ARCHIVES AND RECORDS SERVICE, NATIONAL AUDIO-VISUAL CENTER, ROOM G-5, WASHINGTON, D.C. 20409 (FOR PREVIEW OR SALE)

APPENDIX B

FILM CONTENT AND NARRATION

Film 1. USING A MATHEMATICS LABORATORY APPROACH

On film	Narration
General pan of the mathematics lab with students working on several different projects.	What is a mathematics laboratory? A mathematics laboratory is a setting in which a child, by himself or in a small group, experiences mathematics, its ideas, its processes, and its problems, through actual manipulation of physical objects. In this film you will see children (first, fourth and sixth graders) as they actually work in a mathematics laboratory.
Teacher organizing the class to begin a lab. Graphic display of a conventional classroom.	What does a lab look like and how is it organized? Actually there are many ways to organize a mathematics laboratory. A simple beginning might be a regular classroom with a mathematics corner.
Shelves with materials and instructions.	This corner might contain materials and games with appropriate instructions for their use. Students would choose materials to use at a convenient table or at their desks.

On film	Narration
Graphic display of a mathematics laboratory room.	At the more sophisticated end of the scale, the laboratory might be in a room of its own. It would have individual carrels, and places for small group work as well as a wealth of materials and devices, and instructions for their use.
Graphic display of a modified classroom. Students accessing materials and instructions in some organized manner.	Somewhere between these extremes is the conversion of the classroom into a mathematics laboratory. This means moving furniture to provide for student work stations. It also means providing some organized method for allowing students to access materials.
Girls working on geometric projects - curve stitching - paper folding	What kinds of experiences do mathematics laboratory activities provide? These activities generate in the students the feeling of attacking mathematics independently. Children count, measure, record. These more-or-less play-like activities provide an information-seeking environment--an environment in which children can literally "construct" mathematics. In particular, laboratories provide settings for geometric ideas as related to important mathematical concepts of number and function.

On film	Narration
Boys generating hypotheses about "square" numbers and physically checking them.	Research has shown that children seem to particularly value activities in which the materials not only serve as a basis for ideas, but as a means to check out hypotheses they have made.
Close ups of children physically manipulating material and using directions.	The materials can be anything--classroom junk, cans, boards, paper; the students can construct the needed devices as they go along. The materials may be teacher constructed or commercially obtained. In any case they are for the most part simple.
	The directions, which control the laboratory experience, must allow the student freedom in solving a problem or developing an idea. Yet they must have enough structure to prevent frustration.
	Although they may take many forms, the instructions frequently are found as a set of activity cards, with children progressing through the set at their own rates, in their own patterns, and to whatever extent is appropriate.

On film	Narration
First grade students working on graphing the number of possible paths across a circle joining a varying number of points on the circle.	What are considered desirable qualities in the laboratory experience? A key word in the literature seems to be variety. Another key point is the sequence of activities. One suggested cycle is a period of free play, followed by a game which is structured according to mathematical rules, followed by a period of work with abstractions and practice in applying ideas previously discovered.
Children playing with special blocks on the floor.	Here you see students getting at some basic notions of logic and classification by means of "change-one-thing" or "change-two-things" games.
Boys playing a game and testing logical and classificatory ideas at a table.	Again you see the same game with different materials. This illustrates one kind of variation--presentation of the same ideas using several perceptual variates. That is, the logical ideas were the same in the floor game as in the "face" game on the table.
Children developing a "graph" on a large street grid using minimal written instruction.	Many lab activities have as their basis applications of the processes of measuring and counting. Here are some first grade students using counting to get at certain notions about relations and their graphs of these relations.

On film	Narration
Two boys working on a "street graphing" problem from written instruction.	These fourth grade boys are engaged in a similar activity using other materials and more complex written instructions. This again illustrates the principle of variety, both perceptually and in terms of the depth to which the idea is traced.
Two girls playing games of "jump" and "back-jump" developing clock arithmetic ideas.	Another important consideration is the treatment of an idea in a variety of mathematical settings. In the following scenes you will see two different mathematical settings for exploring the notion of inverse operations and the properties common to operations on numbers. The first scene involves students working on clock arithmetic.
Boys flipping a square to determine the effects of certain moves.	Here the same properties and ideas are explored in the setting of the geometry of symmetry.
Children working with the teacher helping in various settings.	What is the teacher's role in all of this? The teacher must choose and organize the experiences. Although teachers report very few "discipline" problems in the laboratory, they must plan carefully so time is used wisely. More than that, the teacher has to be able to quickly analyze student problems and be ready with appropriate questions and "hints" on an individual basis.

On film	Narration
Two girls working on a geometric number project.	What can teachers expect as outcomes from laboratories? Not all research has shown that laboratories are always superior to other meaningful learning settings in terms of content achievement. However, most students definitely prefer the laboratory setting. In addition, there is evidence of improved attitudes toward mathematics as well as a more experimental outlook toward mathematics.
Two girls making a discovery relating to a version of Euler's formula.	An important by-product of laboratory work is the feeling associated with independent discovery on the part of the student.
	Can mathematics laboratories be more than fun for the students and more than work for the teacher? There is evidence from research to give you encouragement as you develop and use play-like laboratory settings with your students.

Film 2. USING DIAGNOSIS IN A MATHEMATICS CLASSROOM

On film	Narration
Scan group, then focus on: child working continuously child looking puzzled child answering question child playing with toy car child writing correct solution child making many errors	Look at them-- confident . . . and puzzled . . . trying . . . and not trying . . . succeeding . . . and not succeeding . . .
Scan group working	They differ in so many ways-- and we need to consider their individual characteristics and needs as we help them to learn . . .
Fade in on group taking test	We can use many techniques to find out what they know and can do.
Dissolve to closeup of test	We use written diagnostic tests, which focus on specific types of examples and help us to identify specific errors . . .
Fade in on group with teacher, answering questions	But--written diagnostic tests alone are not sufficient: we need to use other techniques, too. We must determine the causes of difficulties that are identified. For instance, we use specific questioning . . .
(Lip Synchronization)	

On film	Narration
Dissolve to closeup of list of questions	These are some of the other questions she asked as the children worked with the example . . .
Fade in on individual being interviewed by teacher	We also use intensive interviewing with individual pupils, to identify specific thinking patterns . . .
(Lip Synchronization)	
Dissolve to closeup of inventory Close in on one line of chart	The teacher recorded the responses of each child . . . Notice how the level of attainment varied for one child . . .
Fade in on teacher watching as children work	We must also carefully observe, to identify children who are succeeding and those who are in need of help . . .
Scan teacher moving among pupils	Research has indicated that identification of a pupil's strengths and weaknesses, diagnosis of the causes of difficulty, followed by instruction designed specifically to resolve any of his difficulties, is an especially helpful technique. How do we use what we've found out through testing, questioning, interviewing, and observing?

On film

Narration

Focus on teacher helping one pupil

We know that a systematically planned program is better than only incidental instruction, and that systematically planned programs should be based on pupil needs. At times we meet these needs by working with individual children . . .

Fade in on closeup of chart with needs and names

Or we use what we've learned about their needs to group children. Such grouping should be flexible, to facilitate teaching to meet the needs which some children have in common at a particular time.

The child must be ready to learn new material.

Close in on one section of chart

Diagnosis disclosed that these children are having difficulty with regrouping in subtraction, and need further work before they are ready to go on . . .

Dissolve to small group working with place value

The teacher provides specific remedial materials and instruction in a small group.

She begins by reviewing . . .

(Lip Synchronization)

On film	Narration
(narration under)	Such review of hundreds, tens, and ones is a necessary step for these children. The teacher then moves to work with the example on the board. Notice the specific questions she asks, both to check and to strengthen their understanding . . .
(Lip Synchronization)	
Return to chart, close in on second section Dissolve to small group working with money problem	The children who have adequate skills and understand regrouping are working with applications involving money . . .
(Lip Synchronization)	
(narration over end of previous scene)	Diagnosis aids us as we plan for children as individuals. Differentiated instruction is provided for enrichment as well as remediation. We want to help all children learn, and diagnosis is essential to this process . . .

Film 3. OPERATIONS WITH WHOLE NUMBERS

On film	Narration
Series of short shots: books being passed out	Work with the operations on whole numbers is at the core of the elementary school mathematics program. Readiness for this begins in the pre-school years, as children learn to use one-to-one correspondence . . .
(Lip Synchronization: count for lunch, milk)	
(Lip Synchronization: passing out straws)	
(narration under)	Research has shown that proficiency in counting and work with sets facilitates the learning of addition and subtraction, and most kindergarten and first grade teachers plan activities which will strengthen this background . . .
Fade in on paper with errors	We know from research that the greatest sources of pupil difficulty with all operations are due to lack of knowledge of basic facts and lack of understandings about the operations.

On film	Narration
Fade to lesson in which the relationship of addition and subtraction is stressed	We are constantly aware of the need to teach basic facts more effectively. Some studies have shown that clarifying the interrelationship of the operations facilitates understanding. Thus, these first graders are focusing on the relationship of addition and subtraction . . .
(Lip Synchronization)	
(narration under)	Notice the use of varied materials. Now the teacher has pupils write equations (or math sentences) on the board. She went on to have them use individual beads and make up equations . . .
Fade out/in	
(Lip Synchronization)	
Fade in on closeups of completed pupil work	Practice work is also directed at strengthening awareness of the relationship between the two operations . . .
Fade in on lesson on multiplication with arrays	In this lesson with third graders, several forms of arrays are included as aids to understanding multiplication. Notice the application of the commutative property as they rename the number . . .
(Lip Synchronization)	

On film	Narration
Fade to closeup of examples on board	Algorithms used to record work with operations cause difficulty for many children. The use of multiple approaches, allowing pupils to reach solutions in many different ways, enables each child to find a way he understands, allowing him to work on his own level . . .
	Underlying each of these is the distributive property of multiplication with respect to addition. Use of a diagram also facilitates computation of the product . . .
Focus on: shorts of first graders working on addition and subtraction	There are certain strategies which can be used across all four operations with whole numbers--as well as with other rational numbers. Research shows that, to facilitate achievement, retention, and transfer, instruction in mathematics must be meaningful. The use of materials is an essential base for developing meaning and understanding . . .
shots of third graders working on multiplication	

Film 4. PRACTICING MATHEMATICS SKILLS

On film	Narration
Fade in on small group being asked questions about clock	We all recognize the necessity of providing practice on what we teach. However, it's pointless--even harmful--to practice unless pupils are ready for practice. Here's a teacher checking on what children remember from earlier work on time. It should be obvious that reteaching needs to be done before these pupils would profit from practice on the knowledges and skills involved . . .
(Lip Synchronization)	
Fade in on lesson with decimals	We know from research that new concepts are less apt to be retained without practice, and that periodic review increases retention. We know that practice is necessary for computational accuracy and for developing efficiency in the use of algorithms. We know that we need to provide practice at appropriate times after understanding has been developed. But how does the teacher know when pupils are ready? Here's one teacher reviewing work with children to find the answer to that question . . . are they ready for practice on division with decimals?
(Lip Synchronization)	

On film	Narration
Focus on pupils working, with teacher moving around, helping individuals	The understanding necessary to profit from the practice work was evident from the way pupils answered the question . . . and so practice is given. Notice that although the teacher provided opportunity for self-checking, she still observes and helps certain individuals . . .
Fade to series of shots: first graders doing worksheets	We know from research that practice should <u>follow</u> meaningful instruction. We know that it should be spaced and varied in type and amount. What types of drill and practice may or should be used to meet varied pupil needs? From first grade on, individual paper-and-pencil work is, of course, used . . .
pupils studying flashcards	Individual practice with flashcards may be used . . .
small group playing game	Group games can be used to provide reinforcement--and they're interesting and fun at the same time . . .
child at CAI terminal	Computer-assisted instruction--CAI--can not only present drill and practice, but can also present an individualized program geared to each child's needs. The computer can be used to store and analyze a vast amount of specific information, much more easily than the teacher can do so . . .

On film	Narration
Fade in on teacher using "follow me" exercise	We also know from research that oral as well as written practice should be provided . . .
(Lip Synchronization)	
Fade in on pupils completing written test, followed by immediate checking	Another effective procedure is to review written work and test items immediately, so children know whether their answers are right or wrong, and can correct incorrect responses right away . . .
(Lip Synchronization)	
Fade to series of shots of teacher saying "good," smiling, etc.	And a key element in the reinforcement of all teaching must be mentioned--the word of praise, the smile, the pat on the shoulder . . .

Film 5. PROBLEM SOLVING TECHNIQUES

On film	Narration
Scan group working	Verbal problem solving is an essential component of the elementary school mathematics program. It is designed to be the "bridge" between the mathematical ideas and skills being learned in the classroom and the use of those ideas and skills in real-life situations. Verbal problem solving has attracted more attention from researchers than any other aspect of the mathematics curriculum. Nevertheless, teachers across all grade levels continue to be challenged and perplexed about effective ways to promote verbal problem solving skills. What suggestions does research provide? First and foremost, it seems evident that a systematic program must be developed-- a program in which children are taught a variety of techniques and procedures to use, and given many opportunities to solve problems. There is no <u>one</u> way which is best: but here are some techniques which have been found to be successful . . .

On film	Narration
Fade in on problem on board; move to pupils acting it out	Have pupils act out some problems using actual or representative materials. This requires analysis of what the problem really means, and contributes to understanding of the process involved, as in this sharing situation . . .
(Lip Synchronization)	
Fade in on class	A second very useful technique is to have pupils make drawings and diagrams. These help them to visualize various ways of solving the problem or verifying solutions . . .
(Lip Synchronization)	
Fade to flannelboard	Sometimes the teacher has a child show his diagram on the flannelboard . . .
Dissolve to series of closeups of drawings on board	Frequently, children can solve problems with drawings that otherwise may be too difficult for them to conceptualize and resolve.
Focus on drawing with 8 boxes	Here is one such problem and the diagram which a third grader used to solve it . . .
Focus on drawing of clock	Here a drawing of a clock face is used as a visual referent for the problem . . .

On film	Narration
Focus on drawing with 4 boxes	Other times a diagram helps to verify a solution . . .
Focus on line with school and houses	Sometimes diagrams help children to solve a problem prior to the time they work with a topic. The diagram was drawn for this problem with fractions: "Jack lives 5/6 of a mile from school. Fred's house is 1/6 mile from Jack's house and on Jack's way to school. How far is Fred's house from school?"
Fade in on lesson with math sentences	Math sentences or equations are widely used. They help to focus attention on the mathematical structure of the problem . . .
(Lip Synchronization) (narration under)	Research evidence is inconclusive on which type of equation is best--one which emphasizes action associated with the problem or one which focuses on the operation used to solve the problem. It could be that one of these types is better for some children; the other, for other children . . .

On film	Narration
Fade in on class determining what A-S-D stand for	Problems with no numbers may help to focus attention on the operation to be used. Here the teacher uses a second technique, in which every child can give his answer, while the teacher can rapidly check the response of each . . .
(Lip Synchronization [with M card])	
Fade in on teacher giving directions	At times, the essential parts of verbal problems should be stressed. Here the teacher is having the children cross out all extraneous data and words which they believe are irrelevant to solving the problem. She asks, "What words in the first sentence are needed? Why? Which are not needed? Why?"
Close in on pupil crossing out words Insert closeup of paper	After the children have completed their papers, they will see how well they agree on the essential and non-essential parts of the problem . . .
Fade in on worksheet with mini-problems	Mini-problems are especially helpful for those children who have difficulty with reading. They can get needed practice in solving problems without meeting reading difficulty at the same time . . .

On film	Narration
Fade in on tape recorder	The tape recorder can also be used to present problems to those with reading difficulties . . .
Move to teacher presenting problem	From kindergarten on, some problems should be presented orally. Children are able to tackle ideas which they might not be able to if they had to use written algorithms to solve the problem . . .
(Lip Synchronization)	
Fade in on chart of Great Lakes	Children must be able to read tables of data. Here the teacher has children interpret a chart and develop some simple problems with the data . . .
(Lip Synchronization)	
Fade in on teacher writing question mark	Children can make up problems for an illustration, either orally or in writing. Often children will suggest different problems for the same illustration. A nonverbal presentation can be effective occasionally . . .
(Lip Synchronization)	

On film	Narration
Fade in on chart of camping equipment	Pupils should be encouraged to estimate answers to problems, and to test the reasonableness of answers. Pupils may thus check their solutions to problems: they are not precise checks, but do help pupils to avoid or detect gross errors . . .
(Lip Synchronization)	
Focus on chart listing techniques	Systematic instruction in problem solving is essential--but no <u>one</u> procedure or series of steps will help all children equally well. The focus in the verbal problem solving program is on conceptualization. This focus is best achieved when computational difficulty is minimized. Problem solving techniques are not mutually exclusive. The use of multiple approaches should be encouraged. However, the technique should be appropriate for the problem and for the child . . .

APPENDIX C

SUPPLEMENTARY FILM MATERIALS

Project on Interpreting Mathematics Education Research

Center for Cooperative Research with Schools
302 Rackley Building
The Pennsylvania State University
University Park, Pennsylvania 16802

Using a Mathematics Laboratory Approach

Research findings and common-sense practice are both reflected in this film.
It is intended to be a vehicle for discussion, not a model of instruction . . .

We can't find research on mathematics on which to base all instruction. Similarly, not everything shown in the film is research-based. However, there is no conflict between what is shown in practice, and research. Of course, this does not imply that there are not other approaches that might have been taken: undoubtedly you will see some things that you would elect to do differently if you were the classroom teacher.

Key Viewing Questions:

What does a mathematics laboratory look like and how is it organized?

What kinds of experiences do mathematics laboratory activities provide?

What is the nature of the materials and instructions in a mathematics laboratory?

What are desirable qualities in the laboratory experience?

What can teachers expect as outcomes from laboratories?

Key Points for Discussion and Study:

In addition to the questions above, the following are appropriate for discussion and study:

(1) How does a mathematics laboratory differ from classrooms in which teachers group pupils and utilize manipulative materials?

(2) How is a mathematics laboratory related to the ongoing program on instruction?

(a) To what extent would it be used in the instructional process?

(b) How would a lab be used to enrich instruction from the point of view of content? From the point of view of mathematical processes?

(c) How would a lab be used to complement ongoing instructional activities?

(3) How would you begin, and organize, a mathematics laboratory?

(4) What are some of the problems associated with operating a mathematics laboratory?

(5) What is the role of the teacher in a mathematics laboratory?

(6) What does research on mathematics laboratories show?

(7) How is the research on using concrete and/or manipulative materials applied in the mathematics laboratory?

(8) What does research indicate as the role of play in mathematics learning?

Project on Interpreting Mathematics Education Research

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Using Diagnosis in a Mathematics Classroom

Research findings and common-sense practice are both reflected in this film.
It is intended to be a vehicle for discussion, not a model of instruction . . .

We can't find research on mathematics on which to base all instruction. Similarly, not everything shown in the film is research-based. However, there is no conflict between what is shown in practice, and research. Of course, this does not imply that there are not other approaches that might have been taken: undoubtedly you will see some things that you would elect to do differently if you were the classroom teacher.

Key Viewing Questions:

Why is diagnosis important?

What do we diagnose?

How do we diagnose?

When do we diagnose?

Key Points for Discussion and Study

On the back of this page are research-supported statements selected from the film. You might want to use them in these ways:

(A) For discussion

What might be done with elementary school pupils, in a variety of mathematical contexts, to reflect each of these statements?

(B) For further study

On which research studies is each statement based? (You may find helpful the bulletins, "Using Research: A Key to Elementary School Mathematics," developed for this project.)

(1) We use written diagnostic tests, which focus on specific types of examples and help us to identify specific errors.

(2) We must determine the causes of difficulties that are identified.

(3) We use <u>specific</u> questioning.

(4) We also use intensive interviewing with individual pupils, to identify specific thinking patterns.

(5) We must carefully observe, to identify children who are succeeding and those who are in need of help.

(6) We know that a systematically planned program is better than only incidental instruction.

(7) Grouping should be flexible, to facilitate teaching to meet the needs which some children have in common at a particular time.

(8) The child must be ready, mathematically, to learn new material.

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Operations with Whole Numbers

Research findings and common-sense practice are both reflected in this film.
It is intended to be a vehicle for discussion, not a model of instruction . . .

We can't find research on mathematics on which to base all instruction. Similarly, not everything shown in the film is research-based. However, there is no conflict between what is shown in practice, and research. Of course, this does not imply that there are not other approaches that might have been taken: undoubtedly you will see some things that you would elect to do differently if you were the classroom teacher.

Key Viewing Questions:

Why is meaningful instruction important?

What is the role of: multiple approaches?
mathematical sentences?
media and varied materials?
diagnosis?
practice?

Key Points for Discussion and Study

On the back of this page are research-supported statements selected from the film. You might want to use them in these ways:

(A) For discussion

What might be done with elementary school pupils, in a variety of mathematical contexts, to reflect each of these statements?

(B) For further study

On which research studies is each statement based? (You may find helpful the bulletins, "Using Research: A Key to Elementary School Mathematics," developed for this Project.)

(1) Proficiency in counting and work with sets facilitates the learning of addition and subtraction.

(2) The greatest sources of pupil difficulty with all operations are lack of knowledge of basic facts and lack of understandings about the operations.

(3) Clarifying interrelationship among the operations facilitates understanding.

(4) Arrays are aids to understanding multiplication.

(5) The commutative property facilitates learning of basic facts.

(6) Some written algorithms (used to record work with operations) cause difficulty for many children.

(7) The use of multiple approaches, allowing pupils to reach solutions in many different ways, enables each child to find a way he understands.

(8) Use of the distributive property of multiplication with respect to addition facilitates computation of products.

(9) Use of a diagram also facilitates computation of products.

(10) To increase achievement, retention, and transfer, instruction in mathematics must be <u>meaningful</u>.

(11) The use of materials is an essential base for developing meaning and understanding.

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Practicing Mathematics Skills

Research findings and common-sense practice are both reflected in this film.
It is intended to be a vehicle for discussion, not a model of instruction . . .

We can't find research on mathematics on which to base all instruction. Similarly, not everything shown in the film is research-based. However, there is no conflict between what is shown in practice, and research. Of course, this does not imply that there are not other approaches that might have been taken: undoubtedly you will see some things that you would elect to do differently if you were the classroom teacher.

Key Viewing Questions:

Why should children practice?
What should be practiced?
How should practice be given?
When should children practice?

Key Points for Discussion and Study:

On the back of this page are research-supported statements selected from the film. You might want to use them in these ways:

(A) For discussion

What might be done with elementary school pupils, in a variety of mathematical contexts, to reflect each of these statements?

(B) For further study

On which research studies is each statement based? (You may find helpful the bulletins, "Using Research: A Key to Elementary School Mathematics," developed for this Project.)

(1) It's pointless -- even harmful -- to practice unless pupils are ready for practice.

(2) New concepts are less apt to be retained without practice, and periodic review increases retention.

(3) Practice is necessary for computational accuracy and for developing efficiency in the use of algorithms (used to record work with operations).

(4) We need to provide practice at appropriate times after understanding has been developed.

(5) Practice should be spaced and varied in type and amount.

(6) Games can be used to provide reinforcement.

(7) Computer-assisted instruction can present drill and practice, with an individualized program geared to each child's needs.

(8) Oral as well as written practice should be provided.

(9) Written work and tests should be reviewed immediately, so children know whether their answers are right or wrong, and can correct incorrect responses right away.

(10) Children should understand the purpose of checking their work.

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Problem Solving Techniques

Research findings and common-sense practice are both reflected in this film.
It is intended to be a vehicle for discussion, not a model of instruction . . .

We can't find research on mathematics on which to base all instruction. Similarly, not everything shown in the film is research-based. However, there is no conflict between what is shown in practice, and research. Of course, this does not imply that there are not other approaches that might have been taken: undoubtedly you will see some things that you would elect to do differently if you were the classroom teacher.

Key Viewing Questions:

What are helpful techniques to use and teach children to use in attacking and solving problems?

Key Points for Discussion and Study:

On the back of this page are research-supported statements selected from the film. You might want to use them in these ways:

(A) For discussion

What might be done with elementary school pupils, in a variety of mathematical contexts, to reflect each of these statements?

(B) For further study

On which research studies is each statement based? (You may find helpful the bulletins, "Using Research: A Key to Elementary School Mathematics," developed for this Project.)

(1) A systematic program must be developed, in which children are taught a variety of techniques and procedures to use, and given many opportunities to solve problems.

(2) There is no one way (of attacking and solving problems) which is best.

(3) It is helpful to have pupils act out some problems using actual or representative materials.

(4) Frequently, children can solve, with drawings and diagrams, problems that otherwise may be too difficult for them to conceptualize and resolve.

(5) Evidence is inconclusive on which type of equation is best -- one which emphasizes action associated with the problem or one which focuses on the operation used to solve the problem.

(6) From kindergarten on, some problems should be presented orally.

(7) Pupils should be encouraged to estimate answers to problems, and to test the reasonableness of answers.

(8) Systematic instruction in problem solving is essential -- but no one procedure or series of steps will help all children equally well.

(9) The focus in the verbal problem solving program is on conceptualization, and is best achieved when computational difficulty is minimized.

APPENDIX D

JOURNAL ANNOUNCEMENT

USING RESEARCH: A KEY TO ELEMENTARY SCHOOL MATHEMATICS

A series of eleven bulletins which attempt to interpret the findings of research on elementary school mathematics for application in the classroom has been developed as one aspect of a project at The Pennsylvania State University. The project is funded by the Research Utilization Branch, Office of Information Dissemination, United States Office of Education. Included are bulletins on these topics: Attitudes Toward Mathematics, Planning for Instruction, The Teaching-Learning Process, Individualizing Instruction, Instructional Materials and Media, Planning for Research, Addition and Subtraction, Multiplication and Division, Fractions and Decimals, Other Mathematical Topics, and Problem Solving and Related Abilities. A set of five films, which illustrate research findings related to math labs, diagnosis, practice, whole number operations and problem solving are also being developed, for distribution beginning Fall, 1970.

The bulletins were prepared by the Project Director, Marilyn N. Suydam, The Pennsylvania State University, and the Project Consultant, J. Fred Weaver, The University of Wisconsin-Madison. They are available (free of charge) by contacting Dr. Suydam, 302 Rackley Building, The Pennsylvania State University, University Park, Pennsylvania 16802.

APPENDIX E

EVALUATION QUESTIONNAIRE, PHASE I

Project on Interpreting Mathematics Education Research
Center for Cooperative Research with Schools
302 Rackley Building
The Pennsylvania State University
University Park, Pennsylvania 16802

Sometime during the past year, you received a copy of the Final Report for Phase I of the "Interpretive Study of Research and Development in Elementary School Mathematics." We are very interested in your reactions to these materials -- and in finding out how you've used them. We would therefore appreciate it very much if you would complete the following questionnaire, and return this evaluation to us before July 15.

Please feel free to write any additional comments you wish.

Thank you!

Marilyn N. Suydam
Project Director

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1. Which best identifies your position? (check one)

_____ a. college teacher of

_____ (1) mathematics methods courses

_____ (2) mathematics content courses

_____ (3) other courses (please specify): _____

_____ b. other position at college level (please specify): _____

_____ c. principal

_____ d. classroom teacher

_____ e. mathematics coordinator/supervisor

_____ f. curriculum specialist

_____ g. student

_____ (1) undergraduate

_____ (2) graduate

_____ h. other (please specify): _____

2. Do you use the Final Report: (check one)

_____ a. frequently

_____ b. never

_____ c. refer to it occasionally

PLEASE TURN PAGE OVER

3. In general is the Final Report: (check one)

_____ a. very helpful

_____ b. somewhat helpful

_____ c. not helpful

4. Which volume have you used most frequently?

_____ volume 1: Introduction and Summary: What Research Says

_____ volume 2: Compilation of Research Reports

_____ volume 3: Developmental Projects

5. Which volume have you used least frequently?

_____ volume 1: Introduction and Summary: What Research Says

_____ volume 2: Compilation of Research Reports

_____ volume 3: Developmental Projects

6. Have others used your copy of the Final Report?

_____ a. Yes -- Approximately how many? _____

_____ b. No

7. Have you used the Final Report with:

_____ a. pre-service teachers (undergraduate students)

_____ b. graduate students

_____ c. in-service teachers

_____ d. other (please specify): _____

8. How have you used the Final Report?

_____

_____

_____

_____

_____

9. Are the summaries in volume 1:

_____ a. Useful? Why? _____

_____ b. Not useful? Why not? _____

_____ c. Bulletins from Phase II are better? Why? _____

_____

10. If you have used volume 1, briefly explain how:

_____

_____

_____

11. Additional comments on volume 1:

_____

_____

_____

12. Are the annotated lists in volume 2:

_____ a. Useful? Why? _____

_____ b. Not useful? Why not? _____

13. Do you have a copy of previous material (Suydam's dissertation) which volume 2 supplements?

_____ a. Yes

_____ b. No -- Would this be useful to you? (contains 799 annotated and categorized reports, 1900-1965)

_____ Yes

_____ No

14. If you have used volume 2, briefly explain how:

_____

_____

_____

15. Additional comments on volume 2:

_____

_____

_____

16. Are the interviews in volume 3:

_____ a. Useful? Why? _____

_____ b. Not useful? Why not? _____

17. If you have used volume 3, briefly explain how:

_____

_____

_____

18. Additional comments on volume 3:

_____

_____

_____

PLEASE TURN PAGE OVER

19. What suggestions do you have that would have improved the Final Report?

_____

_____

_____

_____

_____

Thank you very much for your help.

Please place the completed questionnaire in the enclosed envelope, and drop it in the mail!

APPENDIX F

EVALUATION QUESTIONNAIRE, PHASE II

Project on Interpreting Mathematics Education Research

Center for Cooperative Research with Schools
302 Rackley Building
The Pennsylvania State University
University Park, Pennsylvania 16802

We'd like to know!

You have received our bulletins on "Using Research: A Key to Elementary School Mathematics." Quite naturally, we're interested in your reactions to these materials -- and in finding out how you've been using them. We would therefore appreciate it very much if you would complete the following questionnaire, and return this evaluation to us before June 30. Needless to say, it's important in determining what, if any, future attempts will be made to disseminate similar materials interpreting research.

Please feel free to write any additional comments you wish.

Thank you!

Marilyn N. Suydam

Marilyn N. Suydam
Project Director

---

1. Which best identifies your position? (check one)

_____a. college teacher
  _____(1) mathematics methods courses
  _____(2) mathematics content courses
  _____(3) other courses (please specify):_____
_____b. other position at college level (please specify):_____
_____c. principal
  _____(1) elementary school (k-6)
  _____(2) junior high school (7-9)
  _____(3) senior high school (10-12)
_____d. classroom teacher
  _____(1) elementary school (k-6)
  _____(2) junior high school (7-9)
  _____(3) senior high school (10-12)
_____e. mathematics coordinator/supervisor
_____f. curriculum specialist
_____g. student
  _____(1) undergraduate
  _____(2) graduate
_____h. other (please specify):_____

2. In general, do you find the bulletins useful?

_____a. decidedly
_____b. somewhat
_____c. not at all

PLEASE TURN PAPER OVER: questions 3-10 are on the reverse side.

3. Please put "M" before the two bulletins in Set A you found most helpful.
Please put "L" before the two you consider least helpful.

_____A-1 Attitudes and Interests
_____A-2 Planning for Instruction
_____A-3 The Teaching-Learning Process
_____A-4 Individualizing Instruction
_____A-5 Instructional Materials and Media
_____A-6 Planning for Research in Schools

4. Please put "M" before the two bulletins in Set B you found most helpful.
Please put "L" before the two you consider least helpful.

_____B-1 Addition and Subtraction with Whole Numbers
_____B-2 Multiplication and Division with Whole Numbers
_____B-3 Fractions and Decimals
_____B-4 Other Mathematical Topics
_____B-5 Verbal Problem Solving

5. How do you consider the bulletins in general?

_____a. primarily practical
_____b. primarily theoretical

6. How do you consider the readability of the bulletins?

_____a. clear
_____b. ambiguous

7. For your purposes, was the degree of detail or depth:

_____a. adequate
_____b. more than adequate
_____c. less than adequate

8. Do you feel you can put confidence in the content of the bulletins?

_____a. yes
_____b. no
_____c. uncertain

9. Did the bulletins present a fair and valid interpretation of research as you know it?

_____a. yes
_____b. no
_____c. uncertain

10. Did you notice any bias in the selection of studies or findings?

_____a. yes (please specify):_____
_____b. no

11. Are the bulletins appropriate for use by: (check all which apply)

_____a. pre-service teachers (undergraduate students)
_____b. graduate students
_____c. in-service teachers

12. Who reads the bulletins?

_____a. only read them myself
_____b. distribute copies to others
  _____(1) in-service teachers
  _____(2) pre-service teachers (undergraduate students)
  _____(3) graduate students
  _____(4) administrators and supervisors
  _____(5) other college faculty

13. How many people read your personal copies of the bulletins? _____

14. How many copies have you distributed? _____

15. Have you used the bulletins in: (check all which apply)

_____a. pre-service sessions
_____b. in-service sessions
_____c. discussion/study groups
_____d. curriculum committees
_____e. other (please specify):_____
_____

16. Have you usually read:

_____a. the entire bulletin -- for how many bulletins? _____
_____b. only the Overview -- for how many bulletins? _____
_____c. only the Closer View -- for how many bulletins? _____

17. Have you copied (and distributed):

_____a. the entire bulletin -- for how many bulletins? _____
_____b. only the Overview -- for how many bulletins? _____
_____c. only the Closer View -- for how many bulletins? _____

18. Should these bulletins be reprinted and made available at nominal cost for those in the future who would be interested?

_____a. no
_____b. yes
  _____(1) as separate bulletins
  _____(2) as a collection

19. Have you received all eleven of the bulletins?

_____a. yes
_____b. no Please check the ones you're missing: (titles are included on questions 3 and 4)

_____A-1	_____B-1
_____A-2	_____B-2
_____A-3	_____B-3
_____A-4	_____B-4
_____A-5	_____B-5
_____A-6	

NOW -- fold questionnaire in half (both sheets)
-- staple at bottom
-- drop in the mail

THANK YOU!

Did you notice that on the next page is some additional information about research?